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Willem Verhagen

INFLATION TARGETING AND INTEREST RATE POLICY

Proefschrift

ter verkrijging van de graad van doctor aan de Katholieke Universiteit Brabant, op gezag van de rector magnificus, Prof. dr. F.A. van der Duyn Schouten, in het openbaar te verdedigen ten overstaan van een door het college voor promoties aangewezen commissie in de aula van de Universiteit op woensdag 20 juni 2001 om 16.15 uur

door

Willem Hendrikus Verhagen

geboren op 18 december 1972 te Breda

Promotor: Prof dr. S.C.W. Eijffinger

"live as if you were to die tomorrow,
learn as if you were to live forever"

Mahatma Gandhi

Acknowledgements

If someone were to ask me, "why did you decide to write a PhD-thesis?", I must admit it would be very difficult to give a clear answer. Of course, as an economist I should say I simply maximised the expected value of my intertemporal utility function but (fortunately, I believe) real life is not that simple. For one thing, I do not know exactly what arguments should enter this function and, even if I did, their relative weights could change quite frequently (and in a way I could not even predict myself). Moreover, I was even more uncertain about the alternatives facing me at the time as well as the probability distribution of shocks that could hit me along the way. So, taking the other aspects of who I am into account as well, a more honest answer would be that I may very well have embarked on such an exercise implicitly but also relied heavily on my gut feeling and therefore decided to go for it. Well, in the end I am glad I did and the rest of this book provides an account of my journey as an economist over the past four years. However, in these acknowledgements I would like to take the opportunity to both thank the people who helped create the right professional environment as well as the people who have been important to me in my journey through life up to now.

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Willem Verhagen

Tilburg, April 2001

Contents

Chapter 1: Inflation Targeting as a Framework for Monetary Policy: Between Rules and Discretion

1.1 : The Rules Versus Discretion Debate

The question whether or not monetary policy can affect the real economy has been one of the central themes in monetary economics. Clearly, the classical dichotomy (i.e. a strict separation between the nominal and real side of the economy in which the latter is only affected by real variables) will hold in a world in which all agents have access to the same information set and in which prices are perfectly flexible. The supply of goods and services will then be determined by the availability of production factors and the state of technology. Aggregate demand will automatically adjust to absorb all goods and services produced through movements in the general price level. As a result, on the assumption that the demand for money is non-stochastic and stable, a change in the supply of money will cause an instantaneous and equiproportionate change in the general level of prices without affecting real output. In this situation price stability can be achieved by allowing the money stock to grow at the same rate as output supply.

Based on this notion, most economists now agree that money is neutral in the long run in the sense that monetary policy cannot *systematically* affect real variables such as output and unemployment. Nevertheless, changes in the monetary policy stance generally receive a great deal of attention from the public. There are at least two reasons for this. First of all, monetary policy *can* systematically affect the rate of inflation which is something the public *does* care about. In the real world many assumptions underpinning models which feature the classical dichotomy do not hold. For instance, there seems to be a clear positive empirical relationship between the rate of inflation and its variability. Under conditions of imperfect information about the general price level, the latter may diminish the information content of changes in relative prices thus distorting economic decisions (see e.g. Cukierman (1984)). Next, inflation will cause arbitrary transfers of wealth from net lenders to net borrowers and it may not be possible to insure oneself against this. Even if agents try to insulate themselves from the effects of higher and more volatile rates of inflation it is reasonable to assume that the amount of resources they will devote to such hedging activities (e.g. through the use of financial innovations) will be positively related to the rate of inflation as well. These efforts will diminish the pool of resources available for directly productive activities. Also, as non-indexed financial instruments are still widely used, an increase in inflation uncertainty will cause investors to demand a higher risk premium leading to an increase in long-term real interest rates.[1]

The second reason why the public cares about the monetary policy stance is that monetary policy may not be neutral in the short run due to the existence of nominal rigidities and information asymmetries.[2] Therefore, monetary policy can at least in principle be used to stabilise real variables such as unemployment even though the long run values of these variables will be fully determined by real factors.

Given the fact that monetary policy has a considerable impact on the mean rate of inflation and on the variance of several real variables, both of which affect social welfare, the question then becomes: what is the best way to conduct monetary policy? Traditionally, the answer to this question has divided economists into two camps, those who favour a discretionary policy which can actively react to shocks affecting inflation and output and those who advocate non feed-back time-invariant rules. Basically, discretion implies setting monetary policy optimally in every period given the circumstances prevailing in that period and without paying attention to the implication of the current monetary policy stance for future periods. This view was prevalent especially in the 1960's and early 1970's when it was believed that governments should play an active role in achieving optimal macro-economic outcomes. As far as monetary policy is concerned, it was believed that there existed a permanent and stable trade-off between inflation and unemployment. Hence, the policymaker could pick the socially optimal point on the inflation-output frontier and use a large scale carefully specified macro-econometric model of the economy to determine the monetary (and fiscal) policy stance required to reach this point. However, discretionary policy lost a lot of its appeal as a result of the period of stagflation experienced in the late 1970's and early 1980's. This caught

[1] Related to these arguments, another reason why the public cares about the monetary policy stance is that it may have a considerable effect on financial markets and therefore household wealth (even though of course monetary policy cannot *systematically* affect the fundamental value of e.g. stock prices).

[2] For instance, producers may partly interpret an increase in the general price level as an increase in the relative price of their product which will elicit a temporary increase in production (see Lucas (1973)). The same will happen if an increase in inflation temporarily reduces real wages because nominal wage contracts are signed for a few periods ahead.

proponents of this view by surprise since a simultaneous increase in unemployment and inflation is inconsistent with the afore-mentioned Phillips-curve trade-off.

By contrast, economists who favoured non-activist rules generally placed a lot of faith in the ability of the market mechanism to achieve socially satisfactory results and believed that activist government interventions would more often than not be counterproductive. In other words, due to lags between changes in the instrument of policy and its effect on the economy and due to the fact that policymakers are faced with a lot of uncertainty concerning the workings of the economy and the shocks hitting it, they believed that activist policies may actually end up increasing rather than diminishing the variability of real variables. Moreover, they argued that a simple time-invariant rule would act as a nominal anchor inducing low and stable inflationary expectations thereby helping to maintain a low and stable actual rate of inflation.

The insistence on the importance of inflationary expectations for macroeconomic outcomes also provided the theoretical underpinnings for a successful challenge to the Keynesian notion that there is a stable long run trade-off between inflation and unemployment. Instead, Friedman (1968) argued that the Phillipscurve will be vertical in the long-run in the sense that any gain in output obtained from an increase in inflation will only be temporary. For instance, if workers expect the current price level to be equal to the price level in the previous period, any increase in the nominal wage rate stemming from an increase in the money supply will cause their *perceived* real wage rate to go up. As a result, workers will supply more labour. Simultaneously, employers, who are assumed not to be subject to this kind of misperception, can offer what amounts to a lower real wage rate which causes the demand for labour and hence employment to go up as well. However, as soon as workers realise that the price level has risen, they will demand higher nominal wages to restore the real wage that prevailed before the monetary shock. Eventually, this will cause the rate of unemployment to return to what Friedman termed its *natural level*, i.e. the level of unemployment that prevails in the absence of inflationary surprises and which is determined purely by real (or supply side) factors. Consequently, an increase in the money supply will in the long run only induce a permanently higher rate of inflation without the gain of a reduced unemployment rate.

The realisation that inflationary expectations are highly relevant was further enhanced by the rational expectations revolution in macroeconomics (see e.g. Lucas (1973)). In particular, it led economists to emphasise that the central bank's *credibility* in delivering a low and stable rate of inflation is a very important asset. Kydland and Prescott (1977) and Barro and Gordon (1983) showed that monetary policy can be seen as a strategic game between the central bank and wage setters. Insofar as the socially optimal rate of unemployment lies below the natural rate, a policymaker acting under discretion will be faced with a time-inconsistency problem. Since the policymaker is aware of the fact that any policy action which is anticipated by the public will have no real effects she will find it optimal ex ante to announce a rate of inflation which is equal to the socially optimal rate. However, if the public believes this announcement, the policymaker will have an incentive to cheat ex post and engineer a surprise inflation to bring about a decrease in unemployment.[3] Since the public will be aware of the policymaker's incentives and since it has the same information as the policymaker, it will anticipate this increase in inflation when signing nominal contracts. This will cause the economy to end up with an inflationary bias in the sense that inflation will exceed the socially optimal rate without even a temporary reduction in unemployment. In this setting, the credibility of the central bank and social welfare could be significantly improved if the policymaker were to commit to an ironclad and time-invariant fixed inflation rule.

Hence, both from a practical and a theoretical perspective the balance of opinion seemed to tilt in favour of non-feedback rules as an antidote to the high rates of inflation observed in the 1970's and 1980's. One of the most practical proposals in this respect is the fixed money growth rule proposed by Milton Friedman which was used as a framework for disinflation notably in the US under Fed Chairman Paul Volcker and in the UK under Prime Minister Margaret Thatcher. However, this framework was not without problems. From a practical perspective, maintaining the growth rate of some chosen monetary aggregate within pre-announced bands was greatly complicated by large and unpredictable shifts in the demand for money caused by financial liberalisation and financial innovations.[4] Another example of a fixed rule for monetary policy is the adherence to a fixed exchange rate. This strategy worked very well for the small and very open economy of The Netherlands which tied the guilder to the Deutschmark and (after 1983) geared monetary policy entirely towards the maintenance of

[3] At the point where inflation is equal to the socially optimal rate, the marginal cost of a surprise inflation will be zero while the marginal benefit will be strictly positive since the natural rate of unemployment exceeds the socially optimal rate.

[4] This velocity instability problem may have been partly brought about by the intention to adhere strictly to the pre-announced monetary targets which caused this strategy to fall victim to Goodhart's law which states that *'..any statistical regularity will tend to collapse once pressure is put upon it for control purposes..'* (see Goodhart (1989)

the fixed parity. The fact that De Nederlandsche Bank reacted quickly and without hesitation whenever the foreign exchange markets showed any attempt to test the parity resulted in a very high degree of credibility. This, in turn, allowed the Dutch economy to benefit from the Bundesbank's well-established tradition of monetary stability. However, for many other countries which tried to tie their currencies to the Deutschmark the concomitant inability to respond to particular domestic circumstances proved to be an insurmountable problem. Sooner or later this led to doubts about the credibility of the exchange rate arrangement, increases in interest rate differentials and ultimately the abandonment of this kind of monetary policy rule.

Moreover, in the mid 1980's developments in economic theory also challenged the presumption that a non-feedback and time-invariant rule would be optimal from a social point of view. First of all, within the class of models involving the Lucas supply function (where the ability of the policymaker to affect output in the short run depends on whether or not she has private information) economists began to recognise that the policymaker will have an information advantage over the public. This is because nominal wage contracts will remain in place for a prolonged period of time once they have been signed. Effectively, this means that the policymaker will be able to stabilise *unexpected* output shocks that occur after contracts have been signed because she can generate a surprise inflation (see e.g. Rogoff (1985)). As a result, on the assumption that society cares about the variability of output, a situation in which the rate of inflation reacts to unexpected output shocks may be more conducive to social welfare than the maintenance of a fixed time-invariant inflation rule.[5]
Next, the notion that output is demand determined (at least in the short run) was revived by the New-Keynesian school in macroeconomics which provided explicit micro foundations for the existence of nominal rigidities.[6] Apart from the Lucas supply function this led to two other specifications of the economy's aggregate supply relationship which are now widely used in the literature. First of all, the accelerationst Phillipscurve in which price setting is purely backward looking was revived by Svensson (1997b) who used it to analyse the

[5] Rogoff (1985) introduced the notion of private information about supply shocks to show that society gains from appointing a conservative banker (i.e. a central banker who is more inflation averse than society). In particular, in this model the supply shock is drawn from a normal distribution with mean zero and both the central banker and society have an output target which exceeds the natural rate. The commitment solution then involves the absence of an inflationary bias (in the sense the the expected rate of inflation will be equal to the socially optimal rate) and stabilisation of supply shock in accordance with society's preferences. In the absence of a credible commitment mechanism the optimal degree of conservatism will be determined by the balance between credibliity (i.e. the need to reduce the inflationary bias) and flexiblity (i.e. the need to stabilise ouput in accordance with social preferences).

implementation of inflation targeting. Secondly, Calvo (1983) developed a model in which a fixed fraction of firms acting in a monopolistically competitive environment are allowed to reset their prices in any given period. This will cause firms to base their price setting on expected future marginal costs. As a result the current rate of inflation will be determined by the expected rate of inflation in the *next* period and the current output gap. The implications of this New-Keynesian Phillipscurve are surveyed in Clarida, Gali and Gertler (1999). Since monetary policy is generally agreed to be a major determinant of aggregate demand, these developments further enhanced economists' interest in the question to what extent monetary policy should be used to stabilise real variables.

In sum, both in economic theory and in real world policymaking attention has shifted back and forth between discretionary monetary policy and a policy based on rules. Discretion yields flexibility to respond to particular economic circumstances but because of a lack of commitment and its concomitant credibility problems inflationary expectations may be high and volatile. By contrast, non-feedback rules have the advantage of providing a clear anchor for inflationary expectations but do not have any flexibility to respond to idiosyncratic (and possibly severe) shocks which hit the economy from time to time. Consequently, it would seem optimal to design and implement a monetary policy framework which is, in a manner of speaking, 'between rules and discretion', i.e. such that it combines the advantages of both 'extremes' and therefore by definition avoids most of the problems associated with both approaches. The strategy of direct inflation targeting provides such a framework. [7]

1.2: The Implementation of Inflation Targeting

Starting in the late 1980's many countries which had previously experienced severe problems with other monetary policy frameworks began to see direct inflation targeting as means to achieve monetary stability. Although there are differences in the implementation of this regime across these countries, every inflation targeting regime nevertheless to some extent displays three crucial elements. First of all, the legislative authority assigns an explicit quantitative inflation target to the central bank. Next, the central bank is granted instrument

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[6] For an overview see Mankiw and Romer (1991))

[7] For excellent surveys of inflation targeting see Leiderman and Svensson (1995), Haldane (1995) and Bernanke, Lauback, Mishkin and Posen (1998).

independence to achieve this target over the medium term. Finally, the central bank is held accountable to the legislative authority for achieving the inflation target.

1. An Explicit Quantitative Target

Most governments now realise that the only macroeconomic variables monetary policy can affect systematically are nominal variables such as the rate of inflation and the nominal exchange rate. In order to tie down the long run values of these variables any monetary policy regime needs a nominal anchor. In other words, the real general equilibrium of the economy will determine the long-run values of all real variables but in itself does not yield a solution for the long run rate of inflation.[8] As a result, agents need a clear reference point on which their inflationary expectations can be focussed. In a regime of direct inflation targeting the explicit inflation target serves the purpose of providing a simple and clear nominal anchor. Central banks are expected to achieve this target over an appropriately defined horizon. Which horizon is appropriate in this respect will depend on the control lag (i.e. the amount of time it takes before a change in monetary policy feeds through into the rate of inflation) and the extent to which the central bank cares about stabilising real variables around their long-run natural rates. A concern for stability in the real economy may warrant a temporary deviation of the inflation rate from the target but in essence this concern will only determine the speed with which inflation is returned to target after the economy has been hit by a shock. In other words, a successful implementation of inflation targeting will cause the average rate of inflation over a prolonged period of time to be equal to the assigned target. In this respect, an explicit inflation target will introduce a rule-based aspect in monetary policy.

2. Instrument Independence

Of course, the credibility of such an 'inflation target rule' will be vital to its successful implementation. It is widely agreed that this credibility can to a large extent be achieved by granting the central bank instrument independence. In other words, the central bank should have the freedom to set the interest rate to achieve the assigned policy objectives without political interference. However, the fact that most inflation targeting central banks are essentially subjected to an objective function assigned by the government means that they do

[8] Other nominal variables such as the nominal exchange rate and the nominal interest rate basically consist of a real and a nominal part where the latter is directly influenced by the rate of inflation.

not enjoy goal independence. The reason for this is that many countries which implement inflation targeting feel that goal independence would diminish the democratic legitimacy of monetary policy which in itself is after all a public good. Central bank independence has attracted a great deal of attention in the academic literature (for a survey see De Haan and Eijffinger (1996)). One of the most robust results stemming from this literature is that there appears to be a negative correlation between the degree of central bank independence (CBI) and the average rate of inflation.[9] This may be one of the reasons why so many countries experienced an increase in the degree of CBI since the mid 1980's. From a theoretical perspective, CBI may be modeled in several different ways each of which highlights another aspect. For instance, Eijffinger and Hoeberichts (1998) argue that the degree of CBI will be equal to the influence that a conservative central banker can exert on monetary policy relative to the influence exerted by the government (which is assumed to share society's preferences). In this respect they show that there will be a trade-off between independence and conservativeness. Alternatively, Lohmann (1992) argues that the degree of CBI will be positively related to the cost incurred by the government when it decides to override the decision of the conservative central banker. In this model there will be a range of shock realisations for which the central banker can set monetary policy according to her own preferences. However, for relatively extreme values of the shock the central banker will be forced to partially accommodate the government's preferences to avoid being overridden, i.e. outside the region of independence monetary policy will be determined by a convex combination of the conservative central banker's and the government's preferences. Next, quite recently some economists (e.g. Blinder (1998)) have asserted that the essential difference between a politically subservient and an independent central banker is that the latter will not try to push output above the long-run natural rate systematically. In this view, central bank independence can be seen as a situation in which the government assigns a loss function to the central bank which features the stabilisation of inflation around the assigned target, the stabilisation of output *around the natural rate* and some relative weight on both of these sometimes conflicting objectives.[10] Even if politicians realise that there is no long run trade-off between inflation and unemployment, they may nevertheless be tempted to push output

[9] Of course, this correlation need not imply a causal relationship. For instance, it has been argued that both may be the result of the degree to which society cares about low and stable rates of inflation.

[10] As noted by Cukierman (2000) in practice there is a lot of opaqueness concerning the exact value of this relative weight even though central bank laws do provide some indication of the differences between countries. For instance, the fact that the Maastricht Treaty mentions price stability as the only objective for monetary policy whereas the Bank of England Act passed in 1998 mentions price stability *and* the obligation to support the

above the natural rate when seeking re-election. Obviously, this temptation is not present for central bankers whose term in office is not determined by opinion polls. In this approach the credibility of monetary policy is significantly enhanced since the absence of a temptation to push output above the natural rate systematically implies that there will be no average inflationary bias in monetary policy. Hence, the introduction of an explicit inflation target combined with CBI in the sense described by Blinder (1998) will ensure that the economy has a clear and credible nominal anchor for monetary policy.

Moreover, this kind of CBI also allows the beneficial elements of discretion to be introduced in the conduct of monetary policy.[11] To attain the goals assigned to her the central banker will have to take all relevant information into account when setting the interest rate. As a result, the interest rate will have to respond to all shocks hitting the economy. Hence, the rule-like elements embodied in the assigned loss function will be translated into an *endogenous* optimal instrument rule. In this respect Blinder (1998) argues that an instrument independent central banker will (intuitively) follow the Dynamic Programming Approach. Given the long lags in monetary policy, the policymaker will select an entire path of current and future interest rates which is optimal given all the information she has today. If this information set changes in the next period, she will simply select an entire new path for interest rates. In other words, a policy base on the DPA will, on the one hand, be systematic and forward-looking, but it will at the same time also display the flexibility to respond to particular circumstances.

3. Accountability and Transparency

As mentioned before, price stability as well as a low variation of real variables over time are important to social welfare. Therefore, the central bank should be held accountable to Parliament for achieving its assigned goals.[12] According to Cukierman (1999) the concept of accountability has two distinct aspects. Narrow accountability refers to a situation where (as in the case of the ECB) the main objective of monetary policy is the achievement of price stability and where *'..the central bank is responsible for its performance on this objective to a higher authority..'* (see Cukierman (1999, p. 4)). In this respect, the assignment of an explicit quantitative inflation target constitutes an important element of accountability since it

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government's objectives for growth and unemployment may be seen as an indication that the relative weight on output stabilisation in the UK exceeds that of the ECB.

[11] After all, the detrimental effects of discretion are largely connected to the temptation to systematically exploit the inflation-output trade-off.

provides an unequivocal yardstick by which the public can judge the central bank's performance over a prolonged period of time.
Broad accountability, on the other hand, refers to a situation where the central bank clearly has multiple objectives and in which there is a mechanism through which Parliament can in some specifically prescribed situations alter the central bank's trade-off between the inflation and other objectives. Eijffinger, Hoeberichts and Schaling (1999) refer to this concept as 'accountability through final responsibility' and model it in the tradition of Lohmann (1992) as the cost the government will incur when it decides to override the conservative central banker's decision. Both Cukierman (1999) and Eijffinger et al. (1999) argue that the concept of broad accountability implies a trade-off between CBI and its concomitant credibility bonus, on the one hand, and accountability, on the other. In particular, the latter authors show that an increase in the cost of overriding (which increases CBI and reduces accountability through final responsibility) will lower the inflationary bias at the expense of less stabilisation of supply shocks.

Nevertheless, accountability can also be enhanced in ways that do not reduce CBI. For instance, many elements of central bank transparency (i.e. the degree to which the central bank releases private information about its own preferences and/or shocks hitting the economy to the public) will make it easier to monitor the central bank, thereby increasing the degree of accountability. Cukierman (1999) argues that transparency will become more important when the central bank has multiple objectives. This is because in the short-run there may be a trade-off between these objectives. This may warrant a temporary deviation of the rate of inflation from its assigned target in the sense that inflation is not returned to target at the shortest possible horizon. However, observationally such a discrepancy may also arise when the policymaker deviates in some respects from the policy goals announced to the public and, for instance, tries to attain a level of output which exceeds the natural rate. More transparency about the central bank's preferences will then make it easier for the public to ascertain that the observed difference between the inflation rate and its target is indeed caused by a short-run conflict between the democratically assigned and announced objectives rather than by an attempt to cheat them. In this respect, Eijffinger et al (1999) model the degree of transparency as being inversely related the variance of the central banker's preference shocks and show that the inflationary bias will decrease if preferences become more stable over time.

[12] For an elaborate exposition and quantification of the concept of accountability across different countries see De Haan, Amtenbrink and Eijffinger (1999).

However, the academic debate on the optimal degree of transparency is far from settled. On the one hand, Nolan and Schaling (1996), Faust and Svensson (1999) and Geraats (1999) show that more transparency is beneficial to social welfare for much the same reason as stated in Eijffinger et al. However, Cukierman (1999) shows that a regime of limited transparency may be more conducive to social welfare than a regime of full transparency because in the latter the policymaker will relinquish an important information advantage that could be used to stabilise real variables.[13] The differences of opinion regarding the optimal degree of transparency as well as the different particular circumstances facing central banks are to some extent also reflected in the degree to which various central banks release information to the public. For instance, the Bank of England displays a very high degree of transparency through the publication of conditional inflation forecasts, early publication of the minutes of MPC-meetings, regular inflation reports, Parliamentary Select Committee hearings etc. This practice may be explained by the relatively high degree of importance attached to accountability within the British political system, the emphasis on *individual* responsibility of MPC-members for monetary policy performance and the fact that the Bank of England's remit clearly points to output stabilisation as a secondary objective for monetary policy.

On the other hand, the ECB seems to be more inclined to take the view that accountability implies that it should be judged on its ex post record on achieving price stability causing it to display a more limited degree of transparency. This could be explained by the fact that the drafters of the Maastricht Treaty, by giving the ECB some degree of goal independence (in the sense that the *operational* translation of price stability was left to the ECB Governing Council), attached relatively less importance to what Eijffinger et al. (1999) have termed accountability through final responsibility. Within the Euro area this probably makes sense since, unlike in the case of a single country, it is not clear which higher authority within the European Union should give instructions to the ECB. Next, the fact that the Maastricht Treaty only mentions price stability as the objective of monetary policy causes the need for elaborate ex ante explanations of monetary policy to be less pressing. Finally, the ECB argues that early publication of minutes would impair discussion which can be explained from its emphasis on collective rather than individual responsibility and the fact that it may cause national central bank governors to be criticised within their own countries. Nevertheless, the ECB does make a clear effort to increase its transparency in ways it feels will not impair its independence and

[13] Eijffinger et al (1999) also show that increased transparancy will lead to less stabilisation of supply shocks.

credibility (e.g. regular press conferences and interviews and, recently, the publication of forecasts).

1.3: Outline of this Thesis

As indicated by its title, this thesis will address several issues in the implementation of inflation targeting and its implications for how the central bank sets the interest rate. Given our general discussion in the previous sections it is instructive to clearly outline the specific interpretation of inflation targeting used in Chapters 2-5 of this thesis. First of all, we assume that the government decides on the goals of monetary policy. Hence the central bank does not have goal independence but it does have complete instrument independence in the sense that we abstract from mechanisms by which the government can override the central banker's decision ex post. As far as the goals of monetary policy are concerned, the government determines the explicit inflation target, instructs the central bank to stabilise output around the long run natural rate *and* specifies the relative weight the central bank should attach to these two objectives. Hence, throughout Chapters 2-5 it is assumed that the central bank is fully transparent about its objectives. As far as transparency about economic shocks is concerned we will discuss situations in which there is no information asymmetry in this respect (Chapters 2 and 4) and in which the central bank may have private information (Chapters 3 and 5). The reason for this is mainly that Chapters 2 and 4 deal with sticky price models in which output is demand determined. By contrast, Chapters 3 and 5 deal with models in which we implicitly assume that prices in the goods market are flexible. In other words, in these chapters the monetary transmission mechanism involves the Lucas supply function because of which the central bank needs private information to be able to affect output in the short run. When such an information advantage is present in these chapters this can in principle stem from two different sources. First of all, it may arise in the presence of sticky nominal wages because of which the central bank's reaction to unexpected shocks which occur after nominal contracts have been signed will engineer a surprise inflation. Hence, implicitly, it is assumed that the central bank and the public possess the same information *at contracting time*. It has to be said that in this case the central bank can in principle still be completely transparent about shocks hitting the economy. Since nominal wages are *assumed* to be fixed and since the interest rate is set after they have been signed, releasing such information will have no effect on macroeconomic outcomes. This interpretation is most appropriate for Chapter 3. On the other hand, a surprise inflation can also be brought about if nominal wages and interest rates are determined *simultaneously* and if the public and the central bank have access to different information sets. In particular, the central bank may have private information about upcoming supply shocks. In that case the central bank has a choice between revealing this information (thus impairing its ability to affect output in the short run) or maintaining its information advantage. This situation is more applicable to Chapter 5 of this thesis in which we study macroeconomic outcomes under both a full and a limited transparency regime.[14]

In Chapter 6 we will implicitly take this inflation targeting framework as given and study the conduct of foreign exchange interventions in a large open economy in which the nominal exchange rate has relatively little effect on the domestic objectives of policy. In this model, the central bank displays a limited degree of transparency in two distinct ways. First of all, it retains private information about its own preferences concerning exchange rate movements and, secondly, it provides the market with a noisy signal of the actual intervention volume.

The rest of this thesis is divided into three parts each of which will start with a short discussion of the issues at hand. The first part deals with some (largely positive) issues in the implementation of inflation targeting. In particular, we will study the consequences of inflation targeting for the term structure of interest rates (Chapter 2) and we will provide an explanation for and explore the consequences of the well-established practice of interest rate stepping (Chapter 3). The second part discusses the normative issue of the optimal degree of output stabilisation in an inflation targeting regime. To this end, Chapter 4 presents a sticky price model which is amended with uncertainty about the potential level of output. Chapter 5 discusses a model with flexible prices and determines the optimal degree of output stabilisation under a convex Phillipscurve. Part III of this thesis will discuss the conduct of sterilised foreign exchange interventions in an inflation targeting regime in which the policymaker does not wish to allow his exchange rate objective to interfere with his objectives for inflation and output (Chapter 6). Finally, Chapter 7 concludes.

[14] However, we will not deal with the question which of these regimes is optimal but rather take them as given in order to focus on the main issue of this chapter which is the socially optimal degree of output stabilisation.

Part I: Some Issues in the Implementation of Inflation Targeting

A substantial part of monetary economics deals with the question how to translate ultimate policy goals into current instrument levels. Within the specific context of inflation targeting one of the seminal contributions in this respect was made by Svensson (1997b) who argued that because of lags in monetary policy inflation targeting implies *inflation forecast* targeting. In other words, the central bank's inflation forecast, conditional on all information available today and chosen at an appropriate horizon will become the intermediate target of monetary policy. Which horizon is appropriate in this case may depend on several issues, the most important of which are the time lag between changes in the instrument of monetary policy and its effect on inflation and output and the degree to which the central bank seeks to stabilise output around potential. Svensson argues that the conditional inflation forecast can be seen as the ideal intermediate target since it is by definition closely related to the ultimate goal of monetary policy and since it can be influenced instantaneously by the central bank's instrument.

As argued more elaborately in Chapter 1, an appealing feature of this framework is that it can be seen as a framework with both rule-based elements and the discretion to react to new developments in the economy (see also Bernanke et al (1998)). The explicit inflation target itself and the central bank's optimal conditional inflation forecast provide a clear anchor for inflationary expectations. Of course, the forecast may deviate from the target at any given point in time. However, if monetary policy is conducted optimally, there will be no *systematic* deviations between the forecast and the inflation target. Moreover, in practice inflation targeting central banks such as the Bank of England display a large degree of transparency by making a substantial effort to explain why the inflation forecast differs from the target. For instance, the reasons for this may be that the central bank feels that squeezing the effect of inflationary shocks out of the economy at the shortest possible horizon entails too large a cost in terms of output volatility. Alternatively, the central bank may feel that the risks surrounding the central forecast are asymmetric.

Next, in constructing the conditional inflation forecast the central bank will be forced to take all relevant information into account. Technically, this means that the forecast will be translated into an *endogenous* optimal interest rate rule in which the interest rate will be a function of all the determinants of future inflation and output. As a result, shocks to these determinants will ultimately be reflected in the interest rate.

The Svensson (1997b) model provides a succinct description of the implementation of inflation targeting, outlining principles which apply irrespective of the specific circumstances with which different countries are faced. Nevertheless, as indicated in Chapter 1, these specific circumstances as well as different preferences regarding, for instance, the degree of accountability will cause the details of the implementation of inflation targeting to vary across countries. This has led to a large literature which tries to explain these differences and which aims to ascertain how monetary policy should be conducted in the face of particular circumstances. For instance, recent literature discusses how monetary policy should react to exchange rate movements, model uncertainty, different specifications of the economy's aggregate supply relationship etc. In the second part of this thesis we will address two of these particular issues. First of all, in Chapter 2 we will assess the implication of the implementation of inflation targeting for the term structure of interest rates. In this respect we find that variables such as the duration of long-term bonds and the central bank's relative weight on output stabilisation may have very different implications for short and long term interest rates. In Chapter 3 we rationalise the well-established practice of not changing the interest rate in the face of continuously changing circumstances by assuming the central bank suffers a small loss in utility every time it decides to change the interest rate. Subsequently, we relate differences in the size of interest rate step and the expected time period till the next interest rate step to underlying macroeconomic variables. Finally, we show that an inflationary bias may emerge if preferences are asymmetric even though the central banker does not try to push output above the natural rate systematically.

Chapter 2: Inflation Forecast Targeting and the Term Structure of Interest Rates

2.1: Introduction

Since the early 1990's the conduct of monetary policy in many countries has switched to a regime of direct inflation targeting. This change was triggered either as a result of the breakdown of the relationship between money growth rates and inflation (New Zealand and Canada) or because of the disappointment following the use of exchange rates as an intermediate target (United Kingdom, Sweden and Finland). The use of explicit inflation targets derives its theoretical rationale from the fact that they can be used to overcome credibility problems since they can mimick the results of optimal performance incentive contracts (see Walsh (1995) and Svensson (1997a)).[15] However, these theories assume that central banks can instantaneously choose the rate of inflation. Contrary to this assumption, in practice central banks can only affect inflation imperfectly and after a considerable time lag. Virtually all central banks implement monetary policy by setting the *price* at which the banking system's systematic shortage of central bank balances on the interbank money market will be relieved. This gives the central bank near-perfect and instantaneous control over the day-to-day interbank interest rate. From a theoretical perspective this raises the issue as to how explicit inflation targets should be translated into monetary policy instruments. A first contribution to this question was made by Svensson (1997b) who has shown that, because of lags in the transmission process, inflation targeting implies inflation *forecast* targeting. In this analysis the inflation forecast produced by the central bank's structural model of the economy[16] becomes an ideal intermediate target since it is by definition closely related to the ultimate policy goal and since it can be perfectly controlled by the central bank.

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15 For an analysis that looks at the implications of preference uncertainty for the equivalence of linear Walsh (1995) contracts and (quadratic) Svensson (1997a) inflation targets, see Schaling, Hoeberichts and Eijffinger (1998).

16 Bernanke and Woodford (1997) have argued that inflation forecast targeting can only work if the inflation forecast is based on the central bank's own structural model of the economy. They show that responding to private sector forecasts may lead to indeterminacy or non-existence of a rational expectations equilibrium.

Furthermore, the inflation forecast will lead to an endogenous optimal interest rate reaction function which has the same form as the Taylor rule (Taylor (1993)). Also, the past few years have seen a revival of interest in the importance of the term-structure of interest rates for the transmission of monetary policy (e.g. Turnovsky (1989) and Goodfriend (1997)). When short-term inflationary expectations are given, a particular level of the central bank's key interest rate will pin down the short-term real interest rate. According to the expectations hypothesis of the term structure, the current short-term real rate and market expectations concerning future short-term real rates then determine the long real rate. This long-term real rate term will, in turn, affect the determinants of aggregate demand. Recently, research on the term structure has focussed on explanations for the failure of predictive content of long-short spread for future movements in interest rates (McCallum (1994) and Rudebusch (1995)), and on the interaction between the term structure and shifts in the conduct of monetary policy in VAR-models (Fuhrer and Moore (1995), Fuhrer (1996)).

The purpose of this chapter is to incorporate the term structure of interest rates into the Svensson (1997b) inflation forecast targeting framework. To this end, Section 2 presents a model in which monetary policy affects the real economy via the term structure. In Section 3 we derive the endogenous optimal interest rate reaction function. We show that the optimal short-term interest rate will be *more* responsive to the underlying state of the economy as the lifetime of the long-term bond, as measured by its duration, increases. Next, in Section 4 we discuss the implications of inflation forecast targeting for the long-term real interest rate and the long-term expected inflation rate. We show that the long-term nominal interest rate will be *less* responsive to the current state of the economy as a result of an increase in duration. Moreover, the effect of the relative weight on output stabilisation on the responsiveness of the long-term nominal interest rate turns out to be ambiguous. Finally, in Section 5 we examine the implications of inflation forecast targeting for the spread between short- and long-term interest rates. In particular, the optimal conduct of monetary policy will induce a positive relationship between the nominal term spread and future output. It is shown that this relationship will become stronger if either the duration of the long bond increases or if the relative weight on output stabilisation decreases. Section 6 concludes.

2.2: Monetary Policy and the Term Structure

The purpose of this section is to incorporate the term-structure of interest rates in the Svensson (1997b) inflation forecast targeting framework. To this end we assume that the short-term nominal interest rate (i_t) and the long-term nominal interest rate (I_t) are related by the following version of the Pure Expectations Hypothesis (PEH):

$$I_t = (1-k)\sum_{\tau=t}^{\infty} k^{\tau-t} E_t i_\tau \qquad ; \qquad k \equiv \frac{D}{1+D} \tag{2.1}$$

Here I_t represents the nominal yield to maturity on a bond with maturity m (>1) while i_t denotes the nominal yield on a one period bond which is traded on the interbank money market. This means that i_t is under perfect control of the central bank and can therefore be seen as the instrument of monetary policy.

Schiller, Campbell and Schoeholtz (1983) have shown that any finite maturity bond can be approximated by an infinite maturity consol bond provided the (geometric) weights ensure that the duration of the consol is equal to the duration of the finite maturity bond. Assuming the duration of the long bond is constant and equal to D yields the linear approximation in equation **(2.1)**.[17] An increase in duration will cause the weighting pattern to decline less rapidly because of which, relatively speaking, the long-term interest rate will to a greater extent be determined by expected future short-term interest rates.

For our purposes it turns out to be convenient to rewrite **(2.1)** in its equivalent first-order form:

$$I_t = (1-k)i_t + kE_t I_{t+1} \tag{2.2}$$

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17 The concept of duration allows for a comparison between the holding period returns on discount bonds and coupon bonds. The duration of a discount bond is equal to its maturity. Because a coupon bond can be seen as a package of discount bonds each of which has a different maturity, its duration, which is intended to be a measure of the length of time an investor invests his money, will be a weighted average of the maturities of the underlying discount bonds. The weight on each maturity is then the present value of the discount bond using the coupon bond's yield to maturity as a discount rate.

Note that the long and short real interest rates will be equal if the parameter k is equal to zero. In that case the model will collapse into the original Svensson (1997b) model in which there is no distinction between short and long-term interest rates.

Following the well-known Fisher decomposition, the short-term nominal rate can be written as the sum of the real short-term interest rate (r_t) and expected inflation in the next period conditional on the information available in period t ($E_t\pi_{t+1}$):

$$i_t = r_t + E_t\pi_{t+1} \tag{2.3}$$

Plugging equation **(2.3)** into **(2.1)** and rearranging we can decompose the long-term nominal rate into a long term real interest rate (R_t) and a long-term expected inflation rate conditional on the information available in the current period (Π_t^e):

$$I_t = R_t + \Pi_t^e \quad ; \quad R_t = (1-k)\sum_{\tau=t}^{\infty} k^{\tau-t} E_t r_\tau$$

$$\Pi_t^e = (1-k)\sum_{\tau=t}^{\infty} k^{\tau-t} E_t \pi_{\tau+1} \tag{2.4}$$

Following Svensson (1997b) we assume that inflation and output are linked by the following short-term Phillips-curve relationship:[18]

$$\pi_{t+1} = \pi_t + \alpha_1 y_t \tag{2.5}$$

where $\pi_t \equiv p_t - p_{t-1}$, the inflation rate in period t (p_t is the (log of the) price level). The variable y_t represents the (log of the) output gap in period t where potential output has been normalised to zero. Finally, the parameter α_1 measures the slope of the Phillips-curve. The output gap is determined by the following dynamic relationship:

18 This equation can either be seen as representing a situation of purely backward-looking expectations or as a reduced form of the following equation: $\pi_{t+1} = (1-\theta)E_t\pi_{t+1} + \theta\pi_t + \omega y_t$ in which case it holds that $\alpha_1 = \omega/\theta$.

$$y_{t+1} = \beta_1 y_t - R_t + x_{t+1} \tag{2.6}$$

Following Svensson (1997b) we assume that output is serially correlated ($0 \leq \beta_1 < 1$). However, whereas in the Svensson model output is decreasing in the *short-term* real interest rate with a lag of one period, here we assume that next period's output gap is decreasing in the *long-term* real interest rate (R_t). This assumption can be justified on the grounds that the interest rate sensitive components of aggregate demand generally do not depend directly on the day-to-day interbank money market interest rate but rather on the yield on *some* financial asset with a longer maturity.[19] For simplicity we assume that there is only one long-term interest rate in the output equation. Finally, output is increasing in an exogenous demand shock (x_{t+1}) which is also serially correlated and stationary ($0 \leq \beta_2 < 1$):

$$x_{t+1} = \beta_2 x_t + \varepsilon_{t+1} \quad ; \quad \varepsilon_{t+1} \quad N\left(0, \sigma_\varepsilon^2\right) \tag{2.7}$$

Having described the structure of the economy it remains to specify the preferences of the central bank. Monetary policy is conducted by a central bank with an explicit inflation target π^* which aims to minimise deviations of inflation from this assigned target, on the one hand, and fluctuations of output around the natural rate (which is normalised to zero), on the other.[20] Consequently, the central bank will choose a sequence of current and future short-term nominal rates to minimise the following loss function:

$$L^{CB} = E_t \sum_{\tau=t}^{\infty} \delta^{\tau-t} [\frac{1}{2}(\pi_\tau - \pi^*)^2 + \frac{\lambda}{2} y_\tau^2] \tag{2.8}$$

[19] Of course, there will be many interest rates which pertain to debt instruments of both short and long maturities which affect aggregate demand. In this respect, the parameter D can be seen as an indicator of the relative share of long-term private debt in the economy.

[20] As noted by Svensson (1997b) this means that the central bank does have a long run inflation target (π^*) but no long run output target (other than the natural rate of output). In other words, even though the central bank wishes to limit short-term output variability, in the long run its only objective is price stability.

Here λ represents the central bank's relative weight on output stabilisation while the parameter δ (which fulfils $0 < \delta < 1$) denotes the discount factor (i.e. a measure of the policy horizon). The expectation is conditional on the central bank's information set in period t which contains current output (y_t) the current inflation rate (π_t) and the structure of the economy as described by equations **(2.3)** - **(2.7)**.[21]

2.3: Derivation of the Optimal Instrument Rule

Following Svensson (1997b), the model can be solved by dynamic programming. In this respect, next period's conditional expectation for output ($E_t y_{t+1}$) can be regarded as an *indirect* control variable for the central bank. First of all, since next period's expected rate of inflation is predetermined by the Phillips curve **(2.5)**, perfect control over the short-term nominal interest rate (i_t) implies perfect control over the one period real interest rate (r_t). Next, by assumption the central bank is committed to the (time-invariant) loss function specified in equation **(2.8)** and economic agents face no uncertainty about the parameters of this loss function. This means that the central bank's plan for future instrument levels is fully credible in the sense that there is no discrepancy between the central bank's planned path of future short-term interest rates (conditional on all information available today) and the public's perception of plan. In other words, all expected *future* short-term real interest rates (i.e. $E_t r_{t+j}$, j=1,2,...) will be unambiguously pinned down by the expectation that in every future period the central bank will implement monetary policy so as to minimise its loss function.[22]

Consequently, for the central bank it only remains to set the current short real rate (r_t) so as to attain that specific value of current long real rate (R_t) which is optimal from the point of view of minimising its loss function. Hence, in a fully credible inflation targeting regime the

[21] Note that here the central bank is conducting monetary policy from a clear forward looking perspective. This means that - as elegantly stated by Greenspan in his Congressional testimony on 22 February 1995 - "*..monetary policy will have a better chance of contributing to meeting the nation's macroeconomic objectives if we look forward as we act, however indistinct our view of the road ahead...*"

[22] Of course the expectations will be formed by the private sector. However, we do not have to take this into account explicitly since there is no information asymmetry in the model.

central bank will be able to control the long-term real interest rate perfectly.[23] From the dynamic output equation **(2.6)** it can be seen that this implies perfect control over $E_t y_{t+1}$. Hence, as in Svensson (1997b) the central bank's problem can be reformulated as follows:

$$V(E_t\pi_{t+1}) = \underset{E_t y_{t+1}}{Min}\left\{\left[\frac{1}{2}(E_t\pi_{t+1}-\pi^*)^2 + \frac{\lambda}{2}(E_t y_{t+1})^2\right] + \delta E_t \; V(E_{t+1}\pi_{t+2})\right\}$$

subject to **(2.9)**

$$E_{t+1}\pi_{t+2} = \pi_{t+1} + \alpha_1 y_{t+1}$$

As shown in Appendix A, the first-order condition for the minimisation problem in equation **(2.9)** will yield a rule for the central bank's conditional one-to-two year inflation forecast ($E_t\pi_{t+2}$). This conditional inflation forecast thus becomes the central bank's *intermediate target* for monetary policy and can be expressed as follows:

$$E_t\pi_{t+2} = \pi^* + n\left[E_t\pi_{t+1} - \pi^*\right] \quad ; \quad n \equiv \frac{\lambda}{\delta\alpha_1^2\mu + \lambda} \tag{2.10}$$

The reduced-form parameter μ is a function of the parameters α_1, δ and λ (see Appendix A). If the central bank engages in *strict* inflation targeting (i.e. if the relative weight on output stabilisation (λ) is equal to zero), it will set its intermediate target equal to the inflation target. However in the more realistic case of *flexible* inflation targeting, it will allow $E_t\pi_{t+2}$ to adjust gradually towards the assigned inflation target π^*. The speed of adjustment will then depend negatively on the central bank's relative weight on output stabilisation (i.e. $\partial n/\partial\lambda > 0$, see Appendix A).

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23 It may be argued that this is not in line with reality. It is, however, the logical implication of the present framework in which the public believes that the central bank will always react predictably and optimally to economic shocks. This suggests that uncertainty about the preferences of the central bank may be one of the reasons for imperfect control over the long-term real rate interest rate. By this we mean imperfect control at any *given* moment in time. Of course, even if it had perfect control, the central bank would not be able to engineer a *systematic* deviation of the actual long-term real rate from the equilibrium long-term real rate determined by such factors as thrift and productivity. In our model the latter is assumed to be constant and is normalised to zero.

From equations **(2.5)** and **(2.6)** it can be seen that the *actual* one-to-two year inflation forecast given by the structure of the economy will be equal to:

$$E_t\pi_{t+2} = \pi_t + \alpha_1(1+\beta_1)y_t - \alpha_1 R_t + \alpha_1\beta_2 x_t \tag{2.11}$$

In Appendix B we compute an expression for R_t which is based on the assumption that the central bank seeks to attain the optimal intermediate target in each and every period (i.e. **(2.10)** holds for all $\tau \geq t$). Substituting this expression into **(2.11)** we obtain an equation for the actual one-to-two year inflation forecast in terms of period t state variables and the central bank's instrument:

$$E_t\pi_{t+2} = \left[1+\frac{\alpha_1(1-k)}{(1+k\beta_1)}\right]\pi_t - \frac{kn(1-n)}{(1+k\beta_1)}(\pi_t-\pi^*) + \frac{\alpha_1\beta_2(1-k\beta_2)}{(1+k\beta_1)}x_t$$
$$+\frac{\alpha_1}{(1+k\beta_1)}\left[(1+\beta_1)(1+k\beta_1)+\alpha_1(1-k)-k(n(1-n)+\beta_1^2)\right]y_t - \frac{\alpha_1(1-k)}{(1+k\beta_1)}i_t \tag{2.12}$$

Obviously, the central bank will choose i_t such that the one-to-two year conditional inflation forecast in equation **(2.12)** will be equal to the optimal intermediate target specified in equation **(2.10)**. Hence, by combining these two equations we find the following *endogenous* optimal instrument rule which expresses the optimal short-term nominal interest rate as a function of all variables that characterise the current state of the economy (henceforth to be referred to as economic fundamentals):

$$i_t = \pi^* + \left[1+\frac{(1-n)}{\alpha_1(1-k)}(1+k\beta_1-kn)\right](\pi_t-\pi^*) +$$
$$\frac{(1+\beta_1-n)(1+k\beta_1)+\alpha_1(1-k)-k(n(1-n)+\beta_1^2)}{(1-k)}y_t + \frac{\beta_2(1-k\beta_2)}{(1-k)}x_t \tag{2.13}$$

This equation has the same form as the Taylor rule and explicitly allows for an effect of the term structure of interest rates on the optimal instrument rule (see Proposition 2.1 below).

Note that this endogenous interest rate rule will collapse into the Svensson rule if the output gap is directly determined by the central bank's instrument (i.e. if k=0).[24]

Using equation **(2.3)** and the fact that $E_t\pi_{t+1} = \pi_t + \alpha_1 y_t$, the optimal (ex ante) short-term real rate will be:

$$r_t = \frac{(1-n)}{\alpha_1(1-k)}(1+k\beta_1 - kn)(\pi_t - \pi^*) + \frac{(1+\beta_1-n)(1+k\beta_1)-k(n(1-n)+\beta_1^2)}{(1-k)}y_t + \frac{\beta_2(1-k\beta_2)}{(1-k)}x_t \qquad \textbf{(2.14)}$$

The effect of several parameters on the extent to which both the short-term nominal and short-term real interest rate will respond to the indicator variables can be summarised by the following proposition:

Proposition 2.1:

The responsiveness of the short-term interest rate (either nominal (i_t) or real (r_t)) to economic fundamentals will increase if:

1. the duration of the long-term bond (D) increases
2. the relative weight on output stabilisation (λ) decreases
3. the degree of output persistence (β_1) increases

Proof: see Appendix E

The first part of Proposition 2.1 summarises the effect of the term structure on the central bank's optimal reaction function. The interest rate will respond more strongly to economic fundamentals if the lifetime of the long-term bond increases. This is due to a decrease in policy leverage over the long-term interest rate as the latter will now to a greater extent be determined by expected future short rates. However, provided central bank preferences are

[24] In that case we obtain: $i_t = \pi_t + [(1-n)/\alpha_1](\pi_t-\pi^*) + (1+\beta_1+\alpha_1-n)y_t + \beta_2 x_t$ which can easily be shown to be the solution to a particular variant of this model where it holds that $R_t=r_t$.

constant over time, a change in duration will not alter the central bank's optimal intermediate target as expressed in equation **(2.10)**. Therefore, the central bank will have to manipulate its instrument more actively in order to attain the same desired effect on the long-term real interest rate.

The second part of this Proposition is equivalent to Svensson's finding that the extent to which the central bank's instrument will respond to the current state of the economy will decrease as the central bank cares more about short-term output stabilisation. The intuition is that this will reduce the speed with which the central bank plans to return inflation to its target after the economy has been hit by a shock. Hence, the short-term interest rate will respond less strongly to the current economic situation. This result is insensitive to the question whether or not the term structure of interest rates constitutes an important part of the monetary transmission mechanism.

Finally, an increase in output persistence (β_1) will increase the effect of current exogenous shocks (x_t) on next period's output gap (y_{t+1}). To offset this effect, interest rates will have to be manipulated more actively if the central bank is to attain its objectives for output and inflation stabilisation.

2.4: Implications for the Long-Term Interest Rate

This section will study the implications of the optimal monetary policy rule **(2.13)** for the long real rate (R_t) and the long term expected inflation rate (Π_t^e). First of all, substituting equation **(2.14)** into the expression for R_t obtained in Appendix B we can derive:

$$R_t \quad = \quad \frac{(1-n)}{\alpha_1}(\pi_t - \pi^*) \quad + \quad (1+\beta_1 - n)y_t \quad + \quad \beta_2 x_t \qquad \textbf{(2.15)}$$

Proposition 2.2:

The optimal long-term real interest rate (R_t) will be more responsive to economic fundamentals if:

1. the relative weight on output stabilisation (λ) decreases
2. the degree of output persistence (β_1) increases

The proof follows immediately from equation **(2.15)** where we realise that $\partial n/\partial\lambda > 0$. The intuition is that in our model the long term real interest rate is exactly the same as the ex ante real short term interest rate (i_t - $\alpha_1 y_t$ - π_t) obtained in the model where the term structure is absent (i.e. if k=0, see footnote 11). This result should not be surprising since in both models the central bank seeks to attain the *same* value for $E_t\pi_{t+2}$ which implies that in both models $E_t y_{t+1}$ will be the same. The only difference is that $E_t y_{t+1}$ will be *directly* influenced by i_t in the absence of the term structure while in our model the central bank will set i_t such as to attain that specific value of R_t consistent with its intermediate target.

In Appendix C it is shown that, under the optimal rule, long-term expected inflation will be determined as follows:

$$\Pi_t^e \;=\; \pi^* \;+\; \frac{(1-k)}{(1-kn)}(\pi_t - \pi^*) \;+\; \frac{\alpha_1(1-k)}{(1-kn)}y_t \qquad \textbf{(2.16)}$$

From this expression we can infer the following proposition for the extent to which long-term inflationary expectations will react to changes in the underlying state of the economy:

Proposition 2.3:

The long-term expected rate of inflation (Π_t^e) will be more responsive to economic fundamentals if:

1. the duration of the long-term bond (D) decreases
2. the relative weight on output stabilisation (λ) increases

Proof: see Appendix E

An increase in the lifetime of the long-term bond as measured by its duration will cause the rate of inflation expected to prevail over the lifetime of this bond to be less sensitive to current economic fundamentals. This is because expected (one period) inflation rates in the distant future will exert more influence on the long-term expected rate of inflation. For a given expected path of future inflation rates as given by the central bank's optimal target rule **(2.10)** this will cause the long-term expected rate of inflation to be more stable over time. Next, an increase in the relative weight on output stabilisation will decrease the speed with which the central bank will bring the conditional inflation forecast back in line with the assigned target. As a result, current shocks will to a greater extent feed through into expected future one period inflation rates and consequently into the long-term expected inflation rate.

Finally, using equation **(2.4)** we can express the long nominal interest rate as the sum of equation **(2.15)** and **(2.16)**:

$$I_t = \pi^* + \frac{\alpha_1(1-k)+(1-n)(1-kn)}{\alpha_1(1-kn)}(\pi_t - \pi^*) + \frac{\alpha_1(1-k)+(1-kn)(1+\beta_1-n)}{(1-kn)} y_t + \beta_2 x_t \quad \textbf{(2.17)}$$

Using the results obtained in Propositions 2.2 and 2.3 we can summarise the effect of several parameters on the responsiveness of the long-term *nominal* interest rate to economic fundamentals in the following table:

Table 2.1: Effect of several parameters on the sensitivity of the long nominal rate to economic fundamentals

	R_t	Π_t^e	I_t
D	0	< 0	< 0
λ	< 0	> 0	?
β_1	> 0	0	> 0

This table presents the sign of the partial derivative of the variables listed in the first row with respect to the parameters in the first column. First of all, an increase in the duration of the long-term bond will induce the long-term nominal rate to be *less* sensitive to economic fundamentals. Consequently, while an increase in duration will elicit a more vigorous *instrument response*, this increase in central bank activism will impart a greater degree of stability to the interest rate which affects aggregate demand. Under a fully credible inflation targeting regime (i.e. a regime in which there is no uncertainty about the objectives of the central bank) this effect is entirely induced through the effect on long-term inflationary expectations.

Next, the effect of an increase in the relative weight on output stabilisation (λ) on the long-term nominal rate turns out to be ambiguous. On the one hand, a greater concern for output stabilisation will lead the central bank to eliminate the effect of an inflationary shock more gradually which will translate into a less vigorous response of both the current and future expected short real rates to the state of the economy. On the other hand, this decrease in activism will lead to an enhanced effect of this shock on future expected inflation rates which will increase the expected inflation component in the long-term nominal rate.
Finally, the effect of an increase in degree of output persistence (β_1) works entirely through the induced increased responsiveness of current and expected future short real rates.

2.5: The Behaviour of the Term Spread under the Optimal Rule

This section will look at the implication of an inflation targeting regime for the spread between long and short-term interest rates. The real term spread can be computed by subtracting equation **(2.14)** from equation **(2.15)**. For simplicity we will assume that there is no persistence in the exogenous demand shock (i.e. $\beta_2 = 0$):

$$R_t - r_t = -\frac{k(1-n)(1+\beta_1-n)}{\alpha_1(1-k)}(\pi_t - \pi^*) - \frac{k(1+2\beta_1+n[n-(2+\beta_1)])}{(1-k)}y_t \tag{2.18}$$

Next, the *inflation term spread* can be found by subtracting $E_t\pi_{t+1} = \pi_t + \alpha_1 y_t$ from equation **(2.17)**:

$$\Pi_t^e - E_t\pi_{t+1} \quad = \quad - \quad \frac{k(1-n)}{(1-kn)}(\pi_t - \pi^*) \quad - \quad \frac{\alpha_1 k(1-n)}{(1-kn)} y_t \qquad \textbf{(2.19)}$$

Of course, the nominal term spread is now simply the sum of equations **(2.18)** and **(2.19)** so we have: $I_t - i_t = R_t - r_t + \Pi_t^e - E_t\pi_{t+1}$.

The effect of the underlying cyclical position of the economy on the term structure can be summarised by the following proposition:

Proposition 2.4:

If the economy experiences a *boom*, i.e. if $\pi_t > \pi^*$ and $y_t > 0$, the real term spread, the inflation term spread and therefore also the nominal term spread will be *inverted*, i.e. it will hold that: $R_t - r_t < 0$, $\Pi_t^e - E_t\pi_{t+1} < 0$ and $I_t - i_t < 0$ respectively.

Proof: see Appendix E

If the current rate of inflation exceeds the target and if the output gap is positive there will be an increase in the future rate of inflation. The central bank will not allow inflation to deviate *systematically* from its target and, therefore, output will not *systematically* differ from potential. If the central bank cares about output stabilisation (i.e. if $\lambda > 0$) the optimal target rule **(3.2)** implies that the central bank will disinflate the economy gradually. Consequently, expected future short-term real rates will decline gradually towards zero and expected future inflation rates will decline gradually towards the assigned inflation target. [25]

Since the long-term real rate is a weighted average of expected future real short rates and since the long-term inflation rate is a weighted average of expected future one-period

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25 In the steady state it holds that $\pi_t = \pi^*$ and $y_t = 0$. This implies that the steady state values of both the short-term and the long-term real rates are zero and the steady state values of both the one-period and long-term expected rate of inflation are equal to π^*.

inflation rates, both the real and the inflation term structure will be inverted as a result of optimal monetary policy. Furthermore, this will *induce* a positive relationship between both the real and the inflation term spread in period t and the output gap in period t+1. Formally this can be inferred from the expression for Cov[$(I_t\text{-}i_t)y_{t+1}$] = Cov[$(R_t\text{-}r_t)y_{t+1}$] + Cov[$(\Pi_t^e\text{-}E_t\pi_{t+1})y_{t+1}$]. In Appendix D we show that this will be equal to:

$$Cov[(I_t - i_t)y_{t+1}] \qquad = \left(\frac{k(1+2\beta_1 - n)}{(1+n)(1-k)} + \frac{\alpha_1 k(1-n)^2}{(1-n^2)(1-kn)}\right)\sigma_\varepsilon^2 \qquad \textbf{(2.20)}$$

This result lines up with the literature on the effect of monetary policy on future output (e.g. Bernanke and Blinder (1992), Fuhrer and Moore (1995)) and is essentially the consequence of the systematic 'leaning against the wind' policy described in this section.[26] Even though these studies indicate that the term spread predicts future output growth, they also show that are substantial differences between countries. For instance, Smets and Tsatsaronis (1997, p 4) present evidence that *'(...) the correlation between annual output growth and the lagged term spread is higher in Germany than in the United States(...)'*. They attribute part of this difference to the fact that the influence of inflation scares on the US nominal term spread is much more significant than in Germany as a result of the fact that the Bundesbank enjoys a higher degree of credibility. Furthermore, they present evidence that the Bundesbank reacts more vigorously in real terms to various shocks than the US. While inflation scares do not play a role in this model[27] we can investigate some factors which affect the sensitivity of the nominal term spread to current economic indicators. A more vigorous response of the real term spread and/or the inflation term spread to current indicator variables will lead to a stronger relationship between movements in the *nominal* term spread and future output. The results are summarised in the following proposition:

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26 Since in our model this predictive ability is induced by monetary policy it follows that the central bank cannot base its policy on this predictive abillity. In reality, probably both monetary and non-monetary factors influence this relationship but even then the central bank will have to give a structural interpretation to movements in the yield curve. Indeed, following an instrument rule of the form $i_t = \chi(I_t - i_t)$ as proposed by McCallum (1994) may lead to multiple equilibria (see Bernanke and Woodford (1997)).

27 Since the central bank is credibly committed to the loss function, the path of expected future inflation is unambiguously tied down by the optimal target rule **(3.2)**.

Proposition 2.5:

The nominal term spread (I_t-i_t) will react more strongly to economic fundamentals and as a result the covariance between this spread and future output (Cov[(I_t-i_t)y_{t+1}]) will increase if:

1. the duration of the long bond (D) increases
2. the relative weight on output stabilisation (λ) decreases
3. the degree of output persistence (β_1) increases.

Proof: see Appendix E

An increase in the duration of the long bond will induce an increase in the responsiveness of both the real term spread and the inflation term spread. While an increase in duration will not affect the long-term real rate itself, the induced decrease in policy leverage will cause the *short-term real rate* (r_t) to respond *more strongly* to current economic fundamentals. In other words, as far as the real term spread is concerned, the increase in responsiveness to current indicators and the concomitant increase in its predictive ability with respect to future output can be entirely ascribed to 'the short end' of the (real) yield curve.

By contrast, an increase in duration will make 'the long end' of the inflation term structure (Π_t^e) *less* sensitive to current economic fundamentals while the one period expected rate of inflation will not be affected. Both effects will cause the nominal spread to be more responsive to economic fundamentals. The increased responsiveness of the nominal spread to inflationary shocks is also entirely responsible for the increase in the covariance between the nominal term spread and future output. This is because a change in duration will *not* affect the speed of disinflation.[28] The practical implication of this result is that under a fully credible inflation targeting regime the duration of the debt instrument which affects aggregate demand will be one of the determinants of the response of the nominal term spread to economic developments. Because of this, the financial structure of the economy (i.e. the relative extent

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28 In Appendix D it is shown that optimal monetary policy will yield the following reduced form dynamic equation for output: $y_{t+1} = -(1\text{-}n)y_t - ((1\text{-}n)/\alpha_1)\,(\pi_t\text{-}\pi^*) + \varepsilon_{t+1}$. Consequently, the effect of current monetary policy on future output is fully captured by the parameter n in the optimal target rule **(2.10)**.

to which spending depends on long term interest rates) will influence the observed correlation between the term spread and future output growth. Hence, the fact that many VAR-studies (e.g. Estrella and Mishkin (1997), Smets and Tsatsaronis (1997)) indicate that the real term spread seems to be more strongly related to future output in Germany than in the US could also partly be explained by the fact that the financial structure of the German economy incorporates a larger relative share of long-term debt than the US.

Next, a larger relative weight on short-term output stabilisation (λ) implies a more gradual path of disinflation after the economy has been it by an inflationary shock This will be reflected in a 'flatter' inflation term structure. As for the real term structure this means that the current short-term real rate will be lower than it would have been in the presence of a smaller relative weight on output stabilisation. By contrast, since inflation will be eliminated at a lower pace, expected future short-term real rates will be higher than they would have been for a smaller value of λ. On account of these factors the real term structure will be flatter as well. Hence, the nominal term spread will be less sensitive to indicator variables and its predictive value for future output will diminish because of the fact that monetary policy will exert less influence on next period's output gap.

Finally, an increase in the degree of output persistence (β_1) will increase the effect of current inflationary shocks on output in the next period and on inflation two periods into the future. To offset this effect for a given optimal target rule (i.e. for a given planned path of disinflation) the central bank will display a more activist response to current indicator variables. This will be reflected in an increase in the responsiveness of the real term spread with respect to these indicator variables while the inflation term spread will remain unaffected.

2.6: Summary and Concluding Remarks

This paper incorporates the term structure of interest rates into the Svensson (1997b) inflation forecast targeting framework. We assume that aggregate demand is not directly influenced by the central bank's instrument (i.e. the one period nominal interest rate) but rather by the real yield to maturity on a long-term bond. According to the Pure Expectations Hypothesis (PEH), the *nominal* yield to maturity on this bond will be equal to a weighted average of expected future instrument levels where the weights are a decreasing function of time. The weights can be expressed as a function of the lifetime of the bond as measured by its duration.
Using the time-honoured Fisher decomposition, the model allows us to assess the effect of inflation targeting on the short-term interest rate, the long-term real interest rate and the long-term expected rate of inflation. An increase in the duration of the long bond will increase the responsiveness of the central bank's instrument to the current state of the economy both in nominal and in real terms. However, we also show that this will cause the nominal *long-term* interest rate (i.e. the interest rate which affects spending) to be *less* sensitive to changes in the state of the economy. This effect arises because an increase in duration will cause long-term inflationary expectations to be less volatile.
The explicit distinction between the instrument of monetary policy and the interest rate in the aggregate demand equation also turns out to be crucial when we examine the effect of a change in the central bank's relative weight on output stabilisation. If the central bank pays more attention to output fluctuations, the short-term interest rate will respond less to changes in the current indicators of future inflation. However, the effect on the *long-term* nominal interest rate is ambiguous. Because an inflationary shock will be eliminated more gradually, the long-term real interest rate will be less sensitive to economic fundamentals. By contrast, since it will take longer for the inflationary effect to be eliminated, the long run expected rate of inflation will become more sensitive to the current state of the economy.

The assumed transmission mechanism in this model implies a positive relationship between the nominal term spread and future output which is induced by the optimal response of monetary policy to the state of the economy. In particular, if the economy experiences a boom, both the real and the inflation term spread will be inverted. As for the first one this is because the central bank will raise the short-term nominal interest rate to curb spending and because short-real rates are expected to fall below their current level in future. The inflation term spread reflects the central bank's desired path towards the assigned inflation target which is fully credible to the public. The positive covariance between the nominal term spread and future output arises because a tightening of monetary policy is assumed to affect output with a one-period lag. We also investigated the parameters that affect the responsiveness of the term spread to current economic fundamentals and the strength of the relationship between the term spread and future output. In general, factors which cause the short-term interest rate to become more responsive to indicator variables will also serve to increase both the sensitivity of the term spread to these variables and the correspondence between movements in the term spread and future output.

One crucial assumption in the model is that the public does not face any uncertainty about the parameters of the central bank's objective function. This means that the planned path of future expected inflation rates is fully credible. Moreover, as a result the central bank will have perfect control over the long-term real interest rate because all expected future short-term interest rates will be pinned down by the market's expectation that the central bank will follow its optimal instrument rule in each and every future period. Since perfect control over long-term interest rates is not observed in the real world, one possibly interesting area for future research would therefore be to assess the implications of uncertainty about the central bank's objective function for the term structure of interest rates.

Appendix A: Derivation of the Optimal Intermediate Target

This appendix provides a brief description of the derivation of the optimal intermediate target. For a more elaborate treatment see Svensson (1997b). First of all, from equation **(2.7)** we realise that the indirect loss function will be of the general form:

$$V(E_t\pi_{t+1}) = \mu_0 + \frac{1}{2}\mu(E_t\pi_{t+1} - \pi^*)^2 \tag{A.1}$$

Here, μ_0 and μ are coefficients which remain to be determined. Next, using equation **(2.7)** in the main text, the first-order condition for the central bank's optimisation problem will read as follows:

$$\lambda E_t y_{t+1} + \alpha_1 \delta E_t\left(\frac{\partial V(E_{t+1}\pi_{t+2})}{\partial E_{t+1}\pi_{t+2}}\right) = 0 \tag{A.2}$$

Using equation **(A.1)** to find an expression for the partial derivative between brackets, we can write:

$$E_t\pi_{t+2} - \pi^* = -\frac{\lambda}{\delta\alpha_1\mu}E_t y_{t+1} \tag{A.3}$$

From equation **(2.4)** the conditional forecast of next period's output will be equal to:

$$E\, y_{t+1} = \frac{1}{\alpha_1}(E_t\pi_{t+2} - E_t\pi_{t+1}) \tag{A.4}$$

Plugging this equation into equation **(A.3)** and rearranging will yield equation **(2.10)** in the main text.

Next, in order to identify the coefficients μ_0 and μ in equation **(A.1)** we realise that using equation **(2.9)** we can compute:

$$\frac{\partial V(E_t\pi_{t+1})}{\partial E_t\pi_{t+1}} = E_t\pi_{t+1} - \pi^* + \delta E_t\left\{\frac{\partial V(E_{t+1}\pi_{t+2})}{\partial E_{t+1}\pi_{t+2}}\frac{\partial E_{t+1}\pi_{t+2}}{\partial E_{t+1}\pi_{t+1}}\right\} \tag{A.5}$$

As far as this equation is concerned we note that from equations **(A.1)** and **(2.4)** respectively, we can derive:

$$\begin{aligned}\frac{\partial V(E_{t+1}\pi_{t+2})}{\partial E_{t+1}\pi_{t+2}} &= \mu(E_{t+1}\pi_{t+2} - \pi^*)\\ \frac{\partial E_{t+1}\pi_{t+2}}{\partial E_{t+1}\pi_{t+1}} &= 1\end{aligned} \tag{A.6}$$

Plugging these equations into **(A.5)** and using the expression obtained for ($E_t\pi_{t+2}$ - π^*) in equation **(2.10)** we can rewrite **(A.5)** as follows:

$$\frac{\partial V(E_t\pi_{t+1})}{\partial E_t\pi_{t+1}} = \left[1 + \frac{\lambda\delta\mu}{\delta\alpha_1^2\mu + \lambda}\right](E_t\pi_{t+1} - \pi^*) \tag{A.7}$$

Identification for the coefficient for ($E_t\pi_{t+1}$ - π^*) yields:

$$\mu = \left[1 + \frac{\lambda\delta\mu}{\delta\alpha_1^2\mu + \lambda}\right] \equiv F(\mu) \tag{A.8}$$

From this equation it can be seen that for $\mu \in [0, \infty)$ it will hold that $F(\mu) \in [1, \lambda/\alpha^2)$. Using this it can be shown that the unique positive solution for μ will be:

$$\mu = \frac{1}{2}\left(1 - \frac{\lambda(1-\delta)}{\delta\alpha_1^2} + \sqrt{\left(1 + \frac{\lambda(1-\delta)}{\delta\alpha_1^2}\right)^2 + \frac{4\lambda}{\alpha_1^2}}\right) \geq 1 \tag{A.9}$$

To prove that $\partial n/\partial\lambda > 0$ we first realise that we can write:

$$n = \frac{1}{\delta\alpha_1^2 \frac{\mu}{\lambda} + 1} \quad ; \quad \frac{\partial n}{\partial\left(\mu/\lambda\right)} = -\frac{\delta\alpha_1^2}{(\delta\alpha_1^2 \frac{\mu}{\lambda} + 1)^2} < 0 \qquad \textbf{(A.10)}$$

Furthermore, using equation **(A.9)** we can compute:

$$\frac{\partial\left(\frac{\mu}{\lambda}\right)}{\partial\lambda} = \frac{1}{2}\left\{\frac{\frac{-4}{\alpha_1^2\lambda^2} - 2\left(\frac{\frac{(1-\delta)}{\alpha_1^2\delta} + \frac{1}{\lambda}}{\lambda^2}\right)}{2\sqrt{\left(\frac{(1-\delta)}{\alpha_1^2\delta} + \frac{1}{\lambda}\right)^2 + \frac{4}{\alpha_1^2\lambda}}} - \frac{1}{\lambda^2}\right\} < 0 \qquad \textbf{(A.11)}$$

Consequently, it will hold that: $\partial n/\partial\lambda = \partial n/\partial(\mu/\lambda) * \partial(\mu/\lambda)/\partial\lambda > 0$.

Appendix B: Derivation of the Long Real Rate under the Optimal Monetary Policy Rule

Leading equation **(2.6)** by one period, using equation **(2.6)** in the resulting expression, taking expectations conditional on the information in period t and rearranging we obtain:

$$E_t R_{t+1} = \beta_1^2 y_t - E_t y_{t+2} - \beta_1 R_t + \beta_2(\beta_1 + \beta_2)x_t \qquad \textbf{(B.1)}$$

Furthermore, by leading equation **(2.5)** one period and taking expectations conditional on the information in period t, $E_t y_{t+2}$ can be expressed as follows:

$$E_t y_{t+2} = \frac{1}{\alpha_1}[\, E_t\pi_{t+3} - E_t\pi_{t+2}\,] \qquad \textbf{(B.2)}$$

Since the central bank will follow its optimal target rule in *every* period we can find an expression for the term between brackets by leading equation **(2.10)** one period and subtracting equation **(2.10)** from the result:

$$E_t\pi_{t+3} - E_t\pi_{t+2} = -n(1-n)\,[E_t\pi_{t+1} - \pi^*] \tag{B.3}$$

Using equations **(B.2)** and **(B.3)** we can rewrite **(B.1)** as follows:

$$E_t R_{t+1} = \frac{n(1-n)}{\alpha_1}(\pi_t - \pi^*) + (n(1-n) + \beta_1^2)y_t + \beta_2(\beta_1 + \beta_2)x_t - \beta_1 R_t \tag{B.4}$$

Casting the expression for R_t obtained in equation **(2.4)** in its first-order equivalence (see equation **(2.2)**)and substituting **(B.4)** in the resulting expression yields:

$$R_t = \frac{1-k}{1+k\beta_1}r_t + \frac{kn(1-n)}{\alpha_1(1+k\beta_1)}(\pi_t - \pi^*) + \frac{k(n(1-n)+\beta_1^2)}{1+k\beta_1}y_t + \frac{k\beta_2(\beta_1+\beta_2)}{1+k\beta_1}x_t \tag{B.5}$$

Substituting this expression into equation **(2.11)** using the fact that $r_t = i_t - \pi_t - \alpha_1 y_t$ will yield equation **(2.12)** in the main text.

Appendix C: Derivation of the Long-Term Expected Inflation Rate

Repeated substitution in equation **(2.10)** yields:

$$E_t\pi_{t+i} = \pi^* + n^{i-1}\left[E_t\pi_{t+1} - \pi^*\right] \tag{C.1}$$

Using this in equation **(2.4)** we have:

$$\Pi_t^e \; = \; (1-k)[E_t\pi_{t+1} + k(\pi^* + n(E_t\pi_{t+1} - \pi^*)) + k^2(\pi^* + n^2(E_t\pi_{t+1} - \pi^*)) + ...] \quad \textbf{(C.2)}$$

Solving for the infinite summation and rearranging this can be rewritten as follows:

$$\Pi_t^e \; = \; \pi^* \; + \; \frac{(1-k)}{(1-kn)}[E_t\pi_{t+1} - \pi^*] \quad \textbf{(C.3)}$$

Using the fact that $E_t\pi_{t+1} = \pi_t + \alpha_1 y_t$ we obtain equation **(2.16)** in the main text.

Appendix D: Derivation of the Covariance between the Real and the Inflation Term Spread and Future Output

Plugging the equilibrium solution for R_t obtained in equation **(2.15)** into the dynamic equation for output **(2.6)** and subtracting π^* on both sides of the Phillips-curve relationship **(2.5)** we have the following two-dimensional VAR(1) system:

$$Z_{t+1} \; = \; A\,Z_t \; + \; \phi_{t+1}$$

where: **(C.1)**

$$Z_t \; = \; \begin{bmatrix} y_t \\ \pi_t - \pi^* \end{bmatrix} \quad ; \quad A \; = \; \begin{bmatrix} -(1-n) & -\dfrac{(1-n)}{\alpha_1} \\ \alpha_1 & 1 \end{bmatrix} \quad ; \quad \phi_{t+1} \; = \; \begin{bmatrix} \varepsilon_{t+1} \\ 0 \end{bmatrix}$$

The system has two distinct and real Eigenvalues: $e_1 = 0$ and $e_2 = n$, which indicate that the system is stationary provided n is strictly smaller than one (implying a finite value of λ). Let $vec(\Phi) = (\sigma_\varepsilon^2\ 0\ 0\ 0)^\top$ be the vector form of the variance-covariance matrix of ϕ and let $vec(V) = (\sigma_y^2\ \sigma_{\pi y}\ \sigma_{\pi y}\ \sigma_\pi^2)^\top$ be the vector form of the variance-covariance matrix of Z. Assuming $0 \leq n < 1$ we can compute:

$$vec(V) = (I - A \otimes A)^{-1} vec(\Phi) \quad \Rightarrow \quad vec(V) = \begin{bmatrix} \frac{2\sigma_\varepsilon^2}{(1+n)} \\ \frac{-\alpha_1 \sigma_\varepsilon^2}{(1+n)} \\ \frac{-\alpha_1 \sigma_\varepsilon^2}{(1+n)} \\ \frac{\alpha_1^2 \sigma_\varepsilon^2}{(1-n^2)} \end{bmatrix} \qquad \textbf{(C.2)}$$

From this equation it can be seen that: $\partial\sigma_y^2/\partial n < 0$, $\partial\sigma_{\pi y}/\partial n < 0$ and $\partial\sigma_\pi^2/\partial n > 0$, i.e. λ (and therefore n) affects the trade-off between inflation variability and output variability (see Ball (1997))

The expression for Cov[$(R_t\text{-}r_t)\ y_{t+1}$] can be obtained as follows: First of all, we compute the product of the expression for y_{t+1} found in equation **(C.1)** and the real term spread in equation **(2.19)**. Subsequently, we take the unconditional expectation of the resulting expression where we use the fact that: $E(\pi_t\text{-}\pi^*)^2 = \sigma_\pi^2$, $E(y_t^2) = \sigma_y^2$ and $E((\pi_t\text{-}\pi^*)\ y_t) = \sigma_{\pi y}$ and the fact that ε_{t+1} is exogenous with respect to $(\pi_t\text{-}\pi^*)$ and y_t. Similarly, Cov[$(\Pi_t^e\text{-}E_t\pi_{t+1})y_{t+1}$] can be found by computing the product of equation **(2.19)** and the afore-mentioned expression for y_{t+1} . The summation of these two expression then yields equation **(2.20)** in the main text.

Appendix E: Proofs of Propositions

Proposition 2.1:

Define:[29]

$$a_0 = 1 + \frac{(1-n)(1+k\beta_1 - kn)}{\alpha_1(1-k)} \qquad a_1 = \frac{(1+\beta_1 - n)(1+k\beta_1) + \alpha_1(1-k) - k(n(1-n) + \beta_1^2)}{(1-k)}$$

29 The proof pertains to the reaction coefficients of i_t . However, since $r_t = i_t - E_t\pi_{t+1}$ and since the one period expected inflation rate is predetermined the results carry over to the reaction coefficients of r_t .

$$a_3 = \frac{\beta_2(1-k\beta_2)}{(1-k)}$$

Then it can be shown that

$$\frac{\partial a_0}{\partial k} = \frac{(1-n)(1+\beta_1-n)}{\alpha_1(k-1)^2} > 0 \quad ; \quad \frac{\partial a_1}{\partial k} = \frac{(n-1)^2+\beta_1(2-n)}{(k-1)^2} > 0$$

$$\frac{\partial a_3}{\partial k} = \frac{\beta_2(1-\beta_2)}{(k-1)^2} > 0$$

Since it holds that $\partial k/\partial D = 1/(1+D)^2 > 0$ it follows that $\partial a_i/\partial D > 0$ for i = 0,1,2

Next, as far as the parameter n is concerned, we can compute:[30]

$$\frac{\partial a_0}{\partial n} = -\frac{1+k(1+\beta_1-2n)}{\alpha_1(1-k)} < 0 \quad ; \quad \frac{\partial a_1}{\partial n} = -\frac{1+k(1+\beta_1-2n)}{(1-k)} < 0$$

In Appendix A it is shown that $\partial n/\partial\lambda > 0$. Therefore, we can conclude: $\partial a_0/\partial\lambda < 0$ and $\partial a_1/\partial\lambda < 0$. Finally, we can show:

$$\frac{\partial a_0}{\partial \beta_1} = \frac{k(1-n)}{\alpha_1(1-k)} > 0 \quad ; \quad \frac{\partial a_1}{\partial \beta_1} = \frac{1+k(1-n)}{(1-k)} > 0$$

Proposition 2.3:

Define:

$$b_0 = \frac{(1-k)}{(1-kn)} \quad ; \quad b_1 = \frac{\alpha_1(1-k)}{(1-kn)}$$

[30] Note that the inequality $1+k(1+\beta_1-2n) > 0$ can be rewritten as $\beta_1 > 2n - (1/k) - 1$. The RHS of this expression is strictly increasing in both k and n. Since in addition it holds that $0 \leq k \leq 1$ and $0 \leq n \leq 1$, we know the inequality will always be satisfied if it holds for the specials case in which k=n=1. Substituting this into the inequality yields: $\beta_1 > 0$.

Then it can be shown that:

$$\frac{\partial b_0}{\partial k} = -\frac{(1-n)}{(1-kn)^2} < 0 \quad ; \quad \frac{\partial b_0}{\partial n} = \frac{k(1-k)}{(1-kn)^2} > 0$$

$$\frac{\partial b_1}{\partial k} = -\frac{\alpha_1(1-n)}{(1-kn)^2} < 0 \quad ; \quad \frac{\partial b_1}{\partial n} = \frac{\alpha_1 k(1-k)}{(1-kn)^2} > 0$$

The proof then follows from the fact that $\partial k/\partial D > 0$ and $\partial n/\partial \lambda > 0$.

Proposition 2.4:

As for equation **(2.18)**, the coefficient for $(\pi_t - \pi^*)$ will be smaller than or equal to zero since $0 \leq n < 1$. Next, for the coefficient for y_t to be negative, the denominator of this coefficient needs to be positive, which will be the case since this condition can be rewritten as follows:

$$\beta_1(2-n) > -1-n^2+2n \quad \Leftrightarrow \quad \beta_1 > \frac{-(n-1)^2}{(2-n)}$$

As for equation **(2.19)**, the proof follows immediately from noting that both k an n are restricted to lie within the unit interval. Since the proof holds for the real and inflation term spread it must also hold for the nominal term spread.

Proposition 2.5:

As far as equation **(2.18)** is concerned define:

$$c_0 = \frac{k(1-n)(1+\beta_1-n)}{\alpha_1(1-k)} \quad ; \quad c_1 = \frac{k[1+2\beta_1+n(n-(2+\beta_1))]}{(1-k)}$$

Then it can be shown that:[31]

$$\frac{\partial c_0}{\partial k} = \frac{(1-n)(1+\beta_1-n)}{\alpha_1(k-1)^2} > 0 \quad ; \quad \frac{\partial c_0}{\partial n} = -\frac{k(2(1-n)+\beta_1)}{\alpha_1(1-k)} < 0$$

$$\frac{\partial c_1}{\partial k} = \frac{1+2\beta_1+n(n-(2+\beta_1))}{(k-1)^2} > 0 \quad ; \quad \frac{\partial c_1}{\partial n} = -\frac{k(2(1-n)+\beta_1)}{(1-k)} < 0$$

$$\frac{\partial c_0}{\partial \beta_1} = \frac{k(1-n)}{\alpha_1(1-k)} > 0 \quad ; \quad \frac{\partial c_1}{\partial \beta_1} = \frac{k(2-n)}{(1-k)} > 0$$

Since it is shown in Appendix A that $\partial n/\partial\lambda > 0$, we conclude that $\partial c_i/\partial\lambda < 0$ for $i = 0,1$. Similarly, since $\partial D/\partial k > 0$ it will hold that $\partial c_i/\partial D > 0$ for $i = 0,1$.

Next, as far as equation **(2.19)** is concerned, define:

$$d_0 = \frac{k(1-n)}{(1-kn)} \quad ; \quad d_1 = \frac{\alpha_1 k(1-n)}{(1-kn)} = \alpha_1 d_0$$

Then we can compute:

$$\frac{\partial d_0}{\partial k} = \frac{1-n}{1-kn} > 0 \quad ; \quad \frac{\partial d_0}{\partial n} = -\frac{k}{1-kn} < 0$$

Here we realise that $\partial d_1/\partial k = \alpha_1 \partial d_0/\partial k$ and $\partial d_1/\partial n = \alpha_1 \partial d_0/\partial n$.

Next, using equation **(2.20)** we can compute:

$$\frac{\partial Cov[(R_t - r_t)y_{t+1}]}{\partial k} = \frac{(1+2\beta_1-n)\sigma_\varepsilon^2}{(k-1)^2(1+n)} > 0 \quad ; \quad \frac{\partial Cov[(R_t - r_t)y_{t+1}]}{\partial \beta_1} = \frac{2k\sigma_\varepsilon^2}{(1-k)(1+n)} > 0$$

$$\frac{\partial Cov[(R_t - r_t)y_{t+1}]}{\partial n} = -\frac{2k(1+\beta_1)\sigma_\varepsilon^2}{(1-k)(1+n^2)} < 0$$

[31] Note that it holds that: $0 \le k \le 1$; $0 \le n \le 1$; $0 < \beta_2 < 1$ and that the proof that the denominator in the expression for $\partial a_1/\partial k$ is greater than zero is given in the proof of Proposition 2.2.

$$\frac{\partial Cov\left[\left(\Pi_t^e - E_t\pi_{t+1}\right)y_{t+1}\right]}{\partial k} = \frac{\alpha_1(1-n)\sigma_\varepsilon^2}{(1-kn)(1+n)} > 0$$

$$\frac{\partial Cov\left[\left(\Pi_t^e - E_t\pi_{t+1}\right)y_{t+1}\right]}{\partial n} = -\frac{2\alpha_1 k\sigma_\varepsilon^2}{(1-kn)(1+n)^2} < 0$$

Chapter 3: A Theory of Interest Rate Stepping: Inflation Targeting in a Dynamic Menu Cost Model

3.1: Introduction

"...In sum, given that inflation was forecast to be close to the target in two year's time and that the outlook beyond then was highly uncertain, the Committee could sensibly wait to gather more information before concluding that policy needed to be changed..."

Minutes of Monetary Policy Committee Meeting, 5 and 6 August 1998

As a result of the disappointment with monetary targeting and/or fixed exchange rates, many countries have now adopted a regime of (direct) inflation targeting. The use of explicit inflation targets derives its theoretical rationale from the fact that they can overcome credibility problems since they can replicate the results of optimal performance incentive contracts (see Walsh (1995) and Svensson (1997a)). From a theoretical perspective this has also stimulated the research on monetary policy rules which deal with the question how these explicit inflation targets should be translated into monetary policy instruments (see e.g. Taylor (1993,1998), Svensson (1997b) and Haldane (1997)). This literature explicitly recognises the fact that, because of lags in the transmission mechanism, the actual future rate of inflation will not be under direct control of the central bank. Rather, central banks will use their ability to manipulate the (short-term) interest rate to target the expected future inflation rate conditional on all information that is currently available. Consequently, these models also prescribe the appropriate response to a shock to one of the determinants of inflation. In particular, on the assumption that the central bank only cares about inflation stabilisation it should assess the impact of the shock on the conditional inflation forecast and subsequently change the interest rate so as maintain the equality between the conditional inflation forecast and the assigned inflation target.[32] As a result, the optimal conduct of monetary policy implies

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32 For a formal treatment of this point see Svensson (1997c).

that the short-term interest rate will inherit the time-series properties of the determinants of inflation. However, a stylised fact of actual monetary policy making is that central banks do not immediately change the interest rate in response to new information about the state of the economy.[33] Rather, the instrument of monetary policy tends to remain constant in the face of a changing environment and tends to be changed by discrete amounts while the variables which appear in the central bank's reaction function (e.g. inflation and output) change continuously. Following Bhundia and Yates (1997) we will refer to this phenomenon as *interest rate stepping*. It should be emphasised that this is not the same as interest rate *smoothing*. The latter can be defined as the well-established practice of implementing a desired change in the monetary policy stance in a series of small steps in the same direction rather then taking one single large step all at once.

The purpose of this paper is to reconcile interest rate stepping with optimising behaviour on the part of the central bank and to explore the economic implications of the resulting discrete interest rate changes in a continuously changing environment. To this end we introduce a small 'menu' cost which is incurred every time the central bank changes the interest rate. Following the literature on the impact of such costs of decision making on the behaviour of monopolistic price setters (see e.g. Mankiw (1985) and Akerlof and Yellen (1985)), under these conditions it is no longer optimal for the central bank to respond to small deviations from the optimum. Moreover, in a dynamic setting these costs will *induce* the central bank to take the option value of the status quo into account. Obviously, this option value will be irrelevant if action can be taken at no cost since in that case there is nothing to prevent the central bank from keeping inflation equal to the assigned target continuously. Since the cost, once incurred, will not be reversed by an interest change in the opposite direction, there is an incentive for the central bank to wait and see whether or not the economy will move inflation back towards the target of its own accord. As a result, the central bank will allow the inflation rate to fluctuate freely within a certain range.

The chapter proceeds as follows, Section 2 outlines a simple closed economy and provides a number of reasons for the existence of menu costs. In Section 3 we present the solution to the

33 For useful surveys of this phenomenon see Rudebusch (1995), Goodhart (1996) and Bhundia and Yates (1997).

model under three different scenarios; a benchmark case where menu costs are absent, the case where the central bank solves a string of unrelated 'period' problems and finally the case where the central bank explicitly recognises the intertemporal aspect of its problem. Subsequently, we examine the factors which influence the width of the inflation band. Section 4 examines the implications for the dynamics of short-term interest rates in the light of the empirical literature on this subject. Section 5 solves for the expected rate of inflation and assesses under which conditions the economy will suffer from an inflationary bias. Finally, Section 6 concludes.

3.2: A Simple Closed Economy Model

Consider the following economy in continuous time. Aggregate supply (y_t^s) is given by the familiar Lucas-supply function .

$$y_t^s = \beta(\pi_t - \pi^e) \qquad \textbf{(3.1)}$$

In this equation the natural rate of output (y^*) has been normalised to zero. The parameter β measures the slope of the Lucas supply function, π_t is the (instantaneous) rate of inflation rate and π^e denotes inflationary expectations. As indicated by the absence of a time subscript inflationary expectations do not depend on any particular point in time. One can think of this as the result of the existence of fixed nominal wage contracts. More precisely, agents will determine the expected rate of inflation using the long run probability density function of inflation conditional on the central bank's optimal monetary policy.[34] The exact factors which determine π^e will be discussed in Section 3.5. For now we note that the central bank will take inflationary expectations as given when setting the interest rate. Aggregate demand (y_t^d) is modelled as follows:

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34 Formally, let F be the information set available to private agents containing the information they have about optimal monetary policy and let $g(\pi|F)$ be the long run probability density function of inflation conditional on this information set. Then we have:

$$\pi^e = E(\pi \mid F) = \int_{-\infty}^{\infty} \pi g(\pi \mid F) d\pi$$

$$y_t^d = -\alpha(i_t - \pi_t) + \eta_t \tag{3.2}$$

Here i_t is the instrument of the central bank, i.e. the nominal interest rate which expresses the monetary policy stance (e.g. the UK base rate, the US Federal Funds Target or the ECB's repo rate). The parameter α measures the sensitivity of aggregate demand to the ex post real interest rate and η_t is an exogenous demand shock which follows a driftless Brownian motion:

$$d\eta = \sigma dw \tag{3.3}$$

There is no particular economic reason for assuming a continuous time random walk on the demand shock. However, unlike more sophisticated processes (e.g. exhibiting mean-reversion) this assumption will allow us to compute a relatively simple analytic solution to the central bank's problem.

As far as the preferences of the central bank are concerned, it is assumed that there is a basic trade-off between deviations of the rate of inflation from the assigned target (π^*), on the one hand, and costs which are incurred whenever the interest rate is *changed*, on the other. In view of this trade-off the central bank will minimise the following intertemporal loss function:

$$L(\pi) = E\left\{\int_0^\infty e^{-\delta t}(\pi_t - \pi^*)^2 dt + \sum_j Ce^{-\delta t_j} \middle| \pi_0 = \pi\right\} \tag{3.4}$$

Here δ is the central bank's discount rate (which is inversely related to the policy horizon) and t_j denotes the instants where the central bank decides to change in the interest rate. Each time this happens the central bank will incur a cost which is equal to 'C' for which it holds that C is small (i.e. $C \sim h$). Apart from these costs, the central bank is assumed to engage in strict inflation targeting.[35] While this may seem a restrictive assumption since virtually every

35 In reality, the actual future rate of inflation will of course never be under perfect control by the central bank. However,

central bank also cares about output fluctuations (at least around the natural rate of output) it should be emphasised that in our model, which features only demand shocks, inflation stabilisation *implies* output stabilisation.

The presence of a small cost of changing the interest rate in the central bank's loss function can be rationalised on a number of grounds. First of all, the central bank could partly internalise the costs incurred by agents who are bound into fixed nominal interest rate contracts. For instance, Cukierman (1990, pp. 113) argues that the central bank will be "... *concerned with the predictability of interest rates rather than with their level..".* The reason for this resides in the traditional task of the banking system to provide liquidity by transforming short-term liabilities into long-term assets. This implies that the interest rates charged on the asset side of the balance sheet are fixed for relatively long periods while the interest rates paid on the liability side are likely to change every time the official interest rate changes. Stable official interest rates will therefore reduce the probability of an interest rate mismatch .

Secondly, as argued by Crockett (1994) central bankers may also face a 'psychological' cost when they change their minds, for instance since this makes them vulnerable to allegations of inconsistency or incompetence. As argued by Goodhart (1999) this cost is likely to be prohibitive when the need for a change in the monetary policy stance is not very obvious to outside observers (i.e. when inflation or the inflation forecast is close to the target and output is close to potential). In that case, given the random walk nature of news about these variables, there is a considerable chance that an interest change that is optimal today will have to be reversed in the near future. This might give the impression that the central bank is uncertain about the appropriate direction for monetary policy. Moreover, despite a considerable degree of *formal* independence, the central bank may still be under pressure from politicians not to raise interest rates. As a consequence, the central bank will also be reluctant to lower interest rates because once they are lowered it may be 'politically difficult' to increase them again.[36]

Svensson (1997) has shown that inflation targeting implies that the conditional forecast of inflation becomes the intermediate target of monetary policy. The latter *can* of course be perfectly and instantaneously controlled by the central bank.

36 According to Huizinga and Eijffinger (1999) there is also a strategic argument for not changing the monetary policy

Finally, there is an argument related to the way the interbank money market works. The Fed, for instance, announces a target for the Fed Funds Rate. Unpredictable shifts in the demand curve for central bank balances will cause the Fed Funds Rate to fluctuate randomly around this target (this is because the Fed subsequently corrects these shifts through open market operations to maintain the Fed Funds Rate equal to the target on average). If the Fed were to react optimally to every bit of economic news that comes in it would have to change the Fed Funds Target frequently by probably only a few basispoints. Given the afore-mentioned volatility of the actual Fed Funds Rate this would reduce the information value of interest rate changes which presents an incentive to the Fed to economise on the number of steps to be taken.

3.2: Solution under Static and Rational Expectations

3.2.1 No Menu Costs

As a benchmark we will first solve for the equilibrium in the absence of menu costs (C = 0). From equations **(3.1)** and **(3.2)** we can derive the following reduced form for inflation:

$$\pi_t = \frac{\beta}{\beta-\alpha}\pi^e - \frac{\alpha}{\beta-\alpha}i_t + \frac{1}{\beta-\alpha}\eta_t \qquad \textbf{(3.5)}$$

In order to rule out a perverse response of inflation to its determinants we need to assume that $\beta > \alpha$. Obviously, the central bank's intertemporal loss function **(3.4)** will be minimised if it sets i_t so as to ensure that the condition $\pi_t = \pi^*$ holds continuously.[37] Substituting this condition in equation **(3.5)** and solving for i_t yields the following endogenous instrument rule

$$i_t = \pi^* + \frac{\beta}{\alpha}(\pi^e - \pi^*) + \frac{1}{\alpha}\eta_t \qquad \textbf{(3.6)}$$

This equation is very similar to the Taylor rule (Taylor (1993)) in the sense that it expresses

stance in response to every (supply) shock since this will lower inflationary expectations.

37 Following Svensson (1997) this equality is simply the optimal (intermediate) target rule.

the optimal value of the central bank's instrument as a linear function of the determinants of inflation. In particular, the interest rate will inherit the time-series properties of the demand shock and will therefore also follow a driftless Brownian motion. [38] It appears that the afore-mentioned stability condition concerning the ratio of the slope of the Lucas supply function and the interest rate sensitivity of aggregate demand (β/α) implies that the response-coefficient for (π^e-π^*) will be strictly greater than one. This is a well-known and robust condition for stability in the literature on monetary policy rules (see Taylor (1998)).

Plugging the optimal rule **(3.6)** back into the reduced form for inflation **(3.5)** yields: $\pi_t=\pi^*$. Since wage setters know that the central bank will always keep inflation equal to the target they will determine the expected rate of inflation as follows: $\pi^e = E(\pi) = \pi^*$. As a result, the economy will permanently be at the equilibrium where it holds that $\pi_t = \pi^e = \pi^*$ and $y = y^* = 0$.

3.2.2 Positive Menu Costs

If changing the interest rate is costly, it will no longer be optimal to do so if the deviation of the inflation rate from the target is small (in a manner to be made more precise later). In other words, there will be a trade-off between losses arising from deviations of inflation from its target, on the one hand, and losses stemming from interest rate adjustments, on the other. As a starting point for the analysis we will compute the solution for the inflation rate under the condition that the interest rate is kept constant. Immediately after a change in the interest rate (at, say, t=0) the economy will be in a situation where the inflation rate is equal to the target ($\pi_0=\pi^*$). Without loss of generality we normalise the initial value of the demand shock to zero ($\eta_0 = 0$). Inflationary expectations are fixed and equal to π^e. Plugging these parameter values into the optimal instrument rule **(3.6)** yields the following for the nominal interest rate at t=0:

$$i_0 = \pi^* + \frac{\beta}{\alpha}(\pi^e - \pi^*) \tag{3.7}$$

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38 As we will show later, even in the presence of menu costs this rule describes the long-run behaviour of the interest rate. Since the demand shock follows a continuous time random walk and since inflation will be stationary as a result of optimal monteray policy it follows that both the nominal and the ex post real interest rate will be non-stationary.

Substituting this expression into the reduced form equation for inflation **(3.5)** we obtain an expression for π_t which holds as long as the interest rate is maintained at the value specified in equation **(3.7)**. Since we can repeat this procedure for *every* instant the interest rate is changed we can derive the following general expression for the rate of inflation which holds for all periods between interest rate changes:

$$\pi_t - \pi^* \equiv x_t = \theta\varepsilon_t \quad ; \quad \theta \equiv \frac{1}{(\beta - \alpha)}$$

$$d\varepsilon = \sigma dw \quad ; \quad \varepsilon_0 = \varepsilon_{t_j} = \eta_0 = 0 \tag{3.8}$$

Here ε_t is defined as the stochastic shock to the *inflation gap* (x_t). This shock can be thought of as a re-normalised value of the demand shock (η_t). Starting from t=0 the shock to the inflation gap will be equal to the demand shock (i.e. $\varepsilon_t = \eta_t$ for $i_t = i_0$). Now suppose that at $t=\tau$ the central bank decides to change the interest rate. Obviously, the new interest rate will be set so as to bring inflation back to the target (i.e. it will hold that $i_\tau = \pi^* + (\beta/\alpha)(\pi^e - \pi^*) + \eta_\tau/\alpha$). Since at the time of resetting we need to have $\varepsilon_\tau = 0$ in equation **(3.8)**, it will hold that: $\varepsilon_t = \eta_t - \eta_\tau$ for $i_t = i_\tau$. Of course, this normalisation of the demand shock can be applied to *all* instants in which the interest rate is changed (i.e. for all t_j).

Now suppose the central bank ignores the fact that it is dealing with a *dynamic* optimisation problem and simply solves a string of unrelated 'period' optimisation problems instead. In other words, the central bank will treat ε_t as a 'once and-for-all shock' or, equivalently, it has static expectations in the sense that it does not take the stochastic properties of ε_t into account. At each point in time, the central bank will then compare the discounted present value of the flow cost (x_t^2/δ) to the cost of changing the interest rate (C). Hence, under static expectations, the central bank will set i_t according to the optimal rule **(3.6)** if the following condition is met:

$$\frac{\theta^2\varepsilon_t^2}{\delta} > C \quad \Leftrightarrow \quad (\theta\varepsilon_t)^2 = (\pi_t - \pi^*)^2 > s = C\delta \tag{3.10}$$

Consequently, even under static expectations 'menu' costs which are of second-order smallness ($C \sim h^2$) will lead to a range of inaction which is of first-order smallness ($s \sim h$) . As noted by Dixit (1991) it is in this sense that small menu costs produce relatively large effects.

Under rational expectations the central bank will recognise the intertemporal aspect of its problem and will explicitly take the stochastic process driving the demand shock into account. In other words, if the loss stemming from the inflation gap passes the 'static expectations threshold' in equation **(3.10)** it is no longer optimal to change the interest rate and incur the cost of doing so. This is because the central bank has the *option* to wait and see whether or not the economy will move inflation back to the target level of its own accord. Similar to the case where the central bank has 'static' expectations, the optimisation problem boils down to choosing a threshold level for the inflation gap (b) which will trigger a change in the interest rate. On the assumption that the cost of raising the interest rate is equal to the cost of lowering it, the upper and lower threshold levels imply a symmetric band within which inflation is allowed to move according to the process defined in equation **(3.8)**. Moreover, because the cost of changing the interest rate does not depend on the magnitude of the change (i.e. these costs stem simply from the fact that there *is* a change in the interest rate) the inflation gap will be set to zero whenever it hits one of the thresholds.

First of all, to solve the central bank's problem **(3.4)** we now have to translate the stochastic properties of the shock to the inflation gap (ε_t) into stochastic properties for the inflation gap itself (x_t). Applying the rules of stochastic calculus to equation **(3.8)** we can write:[39]

$$dx = \theta \sigma dw \qquad \textbf{(3.11)}$$

Next, we would like to find an expression for the loss function **(3.4)** which can be minimised with respect to the central bank's choice variable (b). We realise that the interest rate will not be changed as long as the inflation gap is strictly within the band. Hence, for any $x \in (-b, b)$ we can express the RHS of equation **(3.4)** by means of the Bellman equation:

39 When $x = g(\varepsilon)$, where ε follows a driftless Brownian (see equation **(3.3)**), it will hold that:

$$L(x) = x^2 dt + e^{-\delta dt} E\left\{ L(x + dx) \right\} \tag{3.12}$$

Expanding the RHS of this equation and using Ito's lemma (see Appendix A) yields a second-order differential equation:

$$\frac{1}{2}\theta^2\sigma^2 L''(x) - \delta L(x) + x^2 = 0 \tag{3.13}$$

In Appendix A it is shown that the general solution to this equation can be expressed as follows:

$$L(x) = \frac{x^2}{\delta} + \frac{\theta^2\sigma^2}{\delta^2} + A(e^{\gamma x} + e^{-\gamma x}) \quad ; \quad \gamma \equiv \frac{\sqrt{2\delta}}{\theta\sigma} \quad ; \quad A = A(b) \tag{3.14}$$

The first two parts on the RHS denote the expected present value of the loss function under the condition that the interest rate is never changed. Consequently, the third term on the RHS captures the value of being able to make interest rate adjustments. In particular, the effect of the threshold level b on the intertemporal loss function L(x) will be fully incorporated in the constant of integration (A).

It now remains to solve for the constant of integration A and the threshold level b simultaneously. Following Dixit (1991,1993) there are two conditions which pin down these parameters. First of all, the Value Matching Condition (VMC) which says that in the optimum the reduction in the value of L(x) obtained by exercising control should equal the cost of changing the interest rate. In other words, the optimal choice of the threshold level implies that there are no discontinuities in the intertemporal loss function (if there were 'discrete jumps' in L(x) for a particular choice of b this choice would obviously not be optimal). Applying this to equation **(3.14)** we obtain:

$$dx = \left[\frac{1}{2} g''(\varepsilon)\sigma^2\right] dt + g'(\varepsilon)\sigma dw$$

$$L(b) - L(0) = C \quad \Leftrightarrow \quad A\left(e^{\gamma b} + e^{-\gamma b} - 2\right) = C - \frac{b^2}{\delta} \tag{3.15}$$

Secondly, the Smooth Pasting Condition (SPC) which requires the graphs of the L(x) and C-functions to meet tangentially at the point where x=b. This can be understood by observing that, for expression **(3.14)** to be minimised, we need the first order condition A'(b)=0. Differentiating the Value Matching condition with respect to b and using this first order condition yields:[40]

$$L'(b) = 0 \quad \Leftrightarrow \quad \gamma A\left(e^{\gamma b} - e^{-\gamma b}\right) = -\frac{2b}{\delta} \tag{3.16}$$

Since both equation **(3.15)** and **(3.16)** are highly non-linear, A and b can generally only be solved numerically. However Dixit (1991) has shown that the solution for b can be approximated analytically (see Appendix A), this yields:

$$b = \left(\frac{6C\sigma^2}{(\beta - \alpha)^2}\right)^{\frac{1}{4}} \tag{3.17}$$

Hence, under rational expectations fourth-order menu costs ($C \sim h^4$) will have a first-order effect on the band of inaction ($b \sim h$). The reason is that under rational expectations the policymaker will take the *option value of the status quo* into account. In particular, when the inflation gap hits the 'static expectations threshold' specified in equation **(3.10)** it is no longer optimal for the central bank to reset the inflation gap back to zero by incurring the small cost equal to C. Instead at this point the central banker will wait for a small amount of time (dt) during which he will receive new information about the state of the economy. More precisely, the central bank will be able to see if the inflation gap moves back towards zero of its own accord.

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[40] Note that this condition proves that barriers will reduce the value of the loss function (relative to the value obtained in the situation where control is never exercised) since equation (16) will only hold for A < 0.

Consequently, the there will be a trade-off between the 'period' flow cost stemming from the inflation gap, on the one hand, and the cost of exercising control *plus* the option value of the status quo, on the other.

This is illustrated in Figure 3.1 in Appendix E which depicts the situation immediately after an interest rate step has been taken. The aggregate supply curve (y^s) is drawn for the situation where $\pi^e=\pi^*$. The demand shock has the effect of shifting the aggregate demand curve (y^d) randomly along the aggregate supply curve. If there are no costs to changing the interest rate (C=0) the central bank will offset each shock so as to preserve the situation where inflation is equal to the target. However, if changing the interest rate is costly, the demand curve will be allowed to shift around until the rate of inflation hits one of the thresholds.

Equation **(3.17)** allows us to examine the effect of structural and preference parameters on the threshold level for the inflation gap:

Proposition 3.1:

The inflation gap threshold (b) will increase if:

1. The cost of changing the interest rate (C) increases
2. The volatility of the demand shock (σ) increases
3. The slope of the Lucas supply function (β) decreases
4. The interest rate sensitivity of aggregate demand (α) increases

The proof of this proposition follows immediately from equation **(3.17)**. Obviously, an increase in the cost of changing the interest rate will induce the central bank to accept a larger inflation gap before taking action. Next, the effect of the volatility of the demand shock, the slope of the Lucas supply function and the interest rate sensitivity of aggregate demand can be understood from the way they affect the volatility of the stochastic process driving the inflation gap as described in equation **(3.11)**. This is because an increase in the volatility of the inflation gap will also increase the option value of the status quo.

First of all, since the inflation gap is driven by the demand shock, an increase in the volatility of the demand shock (σ) will spill over into higher inflation gap volatility. Next, a decrease in the slope of the Lucas supply function (β) will enhance the effect of a given demand shock on inflation since now a larger part of this shock will be absorbed by inflation at the expense of the effect on output. Finally, if aggregate demand becomes more sensitive to the ex post real interest rate (i_t-π_t) this will enhance the well-known 'vicious circle of instability' by which an increase in inflation will increase aggregate demand through the erosion of real interest rates thereby fuelling a further increase in inflation.

3.4: Implications for the Dynamics of Short-Term Interest Rates

The behaviour of the central bank's key interest rate and the implication of this behaviour for longer-term interest rates has been extensively studied in the empirical literature.[41] For instance, Rudebusch (1995) provides a survey of empirical tests of the expectations hypothesis of the term structure of interest rates, the upshot of which is that term spread predicts future movements in interest rates fairly well in the very short-run (up to 1 month) and in the long run (2 years and longer) . The first finding can be attributed to the tendency of many central banks to smooth interest rates (i.e. to implement the required increase or decrease in a series of small steps rather than all at once). The second observation can be explained by the fact that in the long run the level of interest rates will be determined by the central bank's desire to achieve its ultimate monetary policy goals. Since the latter are to a considerable extent known to the public, agents will be able to predict interest rate movements over long horizons with a reasonable degree of accuracy.

However, in the medium run the predictive ability of the term spread is very poor which led many researchers to reject the rational expectations theory of the term structure. Mankiw and Miron (1986) have argued that the lack of predictive ability can be explained by explicitly taking the manner in which the central bank controls interest rates into account. In particular,

41 Recent examples are Rudebusch (1995), Balduzzi, Berola and Foresi (1997) and Balduzzi, Bertola, Foresi and Klapper (1998).

they suggest the Fed imparts random walk behaviour to the Federal Funds Target in which case the hypothesis of rational expectations implies precisely that future short-term interest rates should not be predictable. This idea has been extended by Rudebusch (1995), Balduzzi et al (1997) and Balduzzi et al (1998). These authors explicitly model the process generating the central bank's target interest rate by postulating that on any given day within the sample period, there will a relatively small but equal probability of a target change of fixed size in either direction.[42] Moreover, Balduzzi et al (1998) document a new stylised fact, namely that the volatility and persistence of the spread increases with the maturity of the loan. They show that spreads of longer-term (e.g. 3 or 6 month) rates from the target are mainly driven by expectations of future target changes. When a target change takes place, all 'adjustment pressures' will be released. However, immediately thereafter the market starts to receive new information which leads to partial predictability of the next target change. Obviously, in view of the fact that the central bank engages in interest rate stepping, the impact of this information on the spread will increase with the maturity of the debt instrument.

In view of this description of interest rate stepping in the empirical literature it is interesting to investigate the factors which determine the size of the interest rate step, the expected duration till the next target change and the extent to which the next target change is predictable. First of all, in our model interest rate steps will always be of a given and fixed size.[43] This is because the interest rate will be reset at a new optimal level if and only if the inflation gap hits one of the thresholds (i.e. if it holds that $|x_t| = b$). Suppose that starting from t=0, the inflation gap first hits one of the barriers at $t = \tau$. From equation **(3.8)** it can be seen that this implies that $|\varepsilon_\tau| = b\,(\beta-\alpha)$. Plugging this expression into the optimal interest rate rule **(3.6)** yields: $|i_\tau| = \pi^* + (\beta/\alpha)\,(\pi^e-\pi^*) + b(\beta-\alpha)/\alpha$. Subtracting the expression for i_0 obtained in equation **(3.7)** and using the expression for b in equation **(3.17)** will yield the following expression for the absolute value of the interest rate step:

42 This formulation abstracts from interest rate smoothing considerations since these will induce a relatively high probability of a target change in the same direction during the first month after a target change (see Rudebusch (1995)).

43 Rudebusch (1995) shows that in reality the size of the interest rate step is drawn from a discrete probability distribution.

$$|i_\tau - i_0| = \frac{(6C\sigma^2)^{\frac{1}{4}}}{\alpha(\beta-\alpha)^{\frac{1}{2}}} \tag{3.18}$$

Proposition 3.2:

The absolute value of the interest rate step ($| i_\tau - i_0 |$) will increase if:

1. The cost of changing the interest rate (C) increases
2. The volatility of the demand shock (σ) increases
3. The slope of the Lucas supply funtion (β) decreases
4. The interest rate sensitivity of aggregate demand (α) increases for $\beta/\alpha < 3/2$ or decreases for $\beta/\alpha > 3/2$

Proof: see Appendix D.

The intuition is that an increase in C, an increase in σ or a decrease in β will induce an increase in the threshold level (b). Hence, a *larger* interest rate step will be needed when the inflation gap hits one of the barriers. As far as an increase in α is concerned there are two opposing effects. On the one hand this will cause the threshold level (b) to go up. On the other hand, since aggregate demand will be more sensitive to interest rate changes, a smaller step will be needed for any *given* value of the threshold which will tend to decrease the size of the interest rate step. The model predicts that the first effect will dominate if the reaction coefficient for (π^e-π^*) in the optimal interest rate rule **(6)** is 'relatively low' (i.e. for $1 < \beta/\alpha <$ 3/2).

Next, we can investigate the factors which affect the expected period of time that will elapse before the next interest rate step is taken (T(x)) . In Appendix C it is shown that for symmetric threshold levels (-b, b) this is given by:

$$T(x) = \frac{(\beta-\alpha)\sqrt{6C}}{\sigma} - \frac{(\beta-\alpha)^2 x^2}{\sigma^2} \tag{3.19}$$

The following proposition summarises the effect of several model parameters on T(x):

Proposition 3.3:

The expected time period that will elapse before the next interest rate step is taken (T(x)) will increase if:

1. the cost of changing the interest rate (C) increases
2. the slope of the Lucas supply function (β) increases for $|x| < b/\sqrt{2}$ or decreases for $|x| > b/\sqrt{2}$
3. the interest rate sensitivity of aggregate demand (α) decreases for $|x| < b/\sqrt{2}$ or increases for $|x| > b/\sqrt{2}$
4. the volatility of the demand shock (σ) decreases for $|x| < b/\sqrt{2}$ or increases for $|x| > b/\sqrt{2}$

Proof: see Appendix D

An increase in the costs of control (C) will increase the threshold level (b) because of which it will take longer before the inflation gap reaches one of the threshold levels. The result for the parameters β,α and σ is basically the outcome of two opposing forces. On the one hand, an increase in β, a decrease in α and/or a decrease in σ will reduce the volatility of the inflation gap (see equation **(3.11)**). This will increase the expected time period that will elapse before the interest rate is reset for any *given* value of the threshold level (b). However, there is also an indirect effect since a decline in the volatility of the stochastic process driving the inflation gap will reduce the threshold level itself (see Proposition 1). All else equal, this will reduce the average time till the next interest rate step.
Which one of these two effects dominates depends on the current value of the inflation gap (x). The model predicts that a decrease in volatility will increase the expected duration of the current monetary policy stance if the inflation gap is relatively small (i.e. if $|x| < b/\sqrt{2}$). In particular, this will hold for the *average duration between two consecutive interest rate steps* (T(0)) which is equal to the first term on the RHS of equation (19). In the empirical literature the probability of a target change during any given day in the sample period is usually estimated using the empirical frequency of target changes (i.e. the number of target changes

divided by the number of business days in the sample, see e.g. Balduzzi et al (1997)). Consequently, our model identifies some of the factors that determine this probability since the latter will be inversely related to T(0).

Corollary 3.1:

The average duration between consecutive interest rate steps (T(0)) will be *increasing* in the slope of the Lucas supply function (β) and the cost of changing the interest rate (C) and *decreasing* in the interest rate sensitivity of aggregate demand (α) and the volatility of the demand shock (σ).

Finally, to obtain an indication of the predictability of the next interest rate step we can compute the probability that the interest rate will be lowered next time Q(x). Suppose that in general the cost of raising the interest rate (C_h) differs from the cost of lowering it (C_l).[44] This will lead to an optimal range of inaction in the interval (-a, b) where a, b > 0. An asymmetry in the cost technology may arise because of the interaction between the desires of politicians and the central bank. For instance, when the latter is to some extent politically subservient, the cost of raising the interest rate may very well exceed the cost of lowering it. Raising the interest rate is politically unpopular while lowering it may yield electoral benefits. The reverse situation may arise when the central bank wants to assert its independence in the face of politicians clamouring for interest rate cuts. In Appendix C it is shown that for $x \in (-a, b)$, the probability that the interest rate will be decreased when it is reset (Q(x)) is given by:[45]

---

14 This would imply: $$L(\pi) = E\left\{ \int_0^{\infty} e^{-\delta t}(\pi_t - \pi^*)^2 dt + \sum_k C_h e^{-\delta t_k} + \sum_m C_l e^{-\delta t_m} \middle| \pi_0 = \pi \right\}$$

where t_k denotes the instants where the interest rate is raised and the central bank incurs a cost equal to C_h while t_m denotes the instants in which it is lowered yielding a cost equal to C_l. In that case we have two Value Matching Conditions ($L(-a)-L(0)=C_l$ and $(L(b)-L(0)=C_h$) and two Smooth Pasting Conditions ($L'(-a)=L'(b)=0$) to determine the two barriers a and b and the two constants of integration. It can easily be shown that $b'(C_h) > 0$ and that $a'(C_l) > 0$ and therefore $C_h > C_l$ implies $b > a$.

45 Of course the probability of an interest rate increase at the next step is simply the complementary probability: $P(x) = 1 - Q(x)$

$$Q(x) = \frac{b-x}{b+a} \tag{3.20}$$

First of all, from this equation it can be seen that interest rate changes are perfectly anticipated by the time they occur (i.e. Q(-a) = 1 and Q(b)=0). This is because in our model the central bank does not have an information advantage over the public. In particular, this means that there is no uncertainty on the part of private agents concerning the position of the thresholds which allows them to anticipate interest rate changes with certainty the instant before they are implemented. Next, the effect of the cost of raising and the cost of lowering the interest rate (C_h and C_l respectively) on the probability of an interest rate decrease at the next step is summarised by the following proposition:

Proposition 3.4:

> The probability of an interest rate decrease at the next step (Q(x)) will increase if the cost of lowering the interest rate (C_l) decreases and/or if the cost of raising the interest rate (C_h) increases.

Proof: see Appendix D.

The intuition is that the absolute value of the upper threshold will exceed the absolute value of the lower threshold if the cost of raising the interest rate exceeds the cost of lowering it. This means that the probability that the inflation gap will first reach the lower threshold will increase for any given rate of inflation.

3.5: The Effect of Dynamic Menu Costs on Inflationary Expectations

Since the expected rate of inflation is locked into nominal wage contracts it will not respond to short-run fluctuations in aggregate demand and/or any one *particular* interest rate response to these fluctuations. In other words, the expected rate of inflation will be determined by agents' beliefs concerning the long-run characteristics of monetary policy. In particular, they know the preferences of the central bank from which they can deduce the range of inaction and, consequently, the long run probability density function for inflation conditional on the thresholds chosen by the central bank. This, in turn, allows them to compute a rational expectation of inflation.

In Appendix C it is shown that for thresholds –a and b, the long run probability density function for the inflation gap $\phi(x)$ will be as follows:

$$\phi(x) = \begin{cases} \dfrac{2(a+x)}{a(a+b)} & for \quad -a \le x < 0 \\ \dfrac{2}{(a+b)} & for \quad x = 0 \\ \dfrac{2(b-x)}{b(a+b)} & for \quad 0 < x \le b \end{cases} \tag{3.20}$$

This probability density function is depicted in Figure 3.2 in Appendix E. Using this we can compute $\pi^e = E(x) + \pi^*$, where the expected value of the inflation gap ($E(x)$) will be equal to:

$$E(x) = \int_{-a}^{b} x\phi(x)dx = \frac{b^2 - a^2}{3(a+b)} \tag{3.21}$$

From this we can infer the following relationship between the inflationary bias and the costs of raising or lowering the interest rate:

Proposition 3.5:

The economy will suffer from an upward (downward) inflationary bias ($\pi^e \gtrless \pi^*$) if the cost of raising the interest rate exceeds (is smaller than) the cost of lowering it ($C_h \gtrless C_l$).

Proof: see Appendix D

In most models an inflationary bias arises because the policymaker faces a systematic temptation to create surprise inflation once nominal contracts are signed. This is because unanticipated inflation enables the policy maker to pursue various real objectives[46] (e.g. an output level which is higher than the natural output level). In this model the central bank is *not* tempted to cheat the public since its only *ultimate* monetary policy goal is to stabilise inflation. The introduction of a small menu cost does not alter this basic fact, even though it means that control will no longer be exercised continuously. In that case, provided the cost structure is symmetric (implying a = b) inflation will not deviate *systematically* from its target (π^*) because of which the latter will feature as the expected rate of inflation which is locked into nominal wage contracts. All this implies that observationally the economy will move along a stable Phillips curve of the form $\pi_t = \pi^* + (1/\beta)y_t$. This relationship is stable *precisely* because the central bank does not systematically try to take advantage of this relationship[47].

However, if for reasons mentioned earlier the cost of raising the interest rate is higher than the cost of lowering it, the probability mass to the right of the point where the inflation gap is zero ($\phi(0)$) will exceed the probability mass to the left of this point (this is the situation depicted in Figure 3.2). Taking this into account wage setters realise that the tendency to maintain the current policy stance longer in the face of upward inflationary pressures will produce an average rate of inflation which is higher than the target. At the risk of repetition it should be noted that this inflationary bias arises even though the central bank does *not* face a systematic temptation to generate surprise inflation. The optimal instrument rule **(3.6)** is fully credible (in the sense that the public faces no uncertainty about this rule) and implies that the inflation gap will be set equal to zero every time the central bank decides to 'switch this rule on'. Moreover, changes in the monetary policy stance (i.e. the interest rate) are always *fully* anticipated the instant before they occur.

46 For a survey see Cukierman (1992, Chs. 2-7)

47 Of course, if the central bank were to try and take advantage of this relationship it would break down as a result of the Lucas critique. In other words, this stable Phillips curve would fall victim to Goodhart's law that '..any statistical regularity will tend to collapse once pressure is placed upon it for control purposes…' (Goodhart (1989))

3.6: Summary and Conclusion

This paper studies a simple model of inflation targeting in which inflation stabilisation features as the only ultimate goal of monetary policy. In addition, the central bank incurs a small cost every time the monetary policy stance (i.e. the short-term interest rate) is changed. Since this cost will induce the central bank to take the option value of the status quo into account it will have a considerable effect on the inflation outcome. In particular, costs of fourth-order smallness will have a first order effect on the band within which inflation is allowed to fluctuate without a change in the interest rate. This band provides an explanation for the well-documented central bank practice of interest rate stepping. We examine how the width of this band depends on the cost of changing the interest rate and the volatility of the inflation process. The latter will be determined by the volatility of the underlying demand shocks, the slope of the Lucas supply function and the interest rate sensitivity of aggregate demand.

In the empirical literature interest rate stepping has been used extensively to offer a 'rational expectations consistent' explanation for the failure of the term spread to predict future movements in short-term interest rates. In view of these results we assessed the factors that determine the size of the interest rate step, the expected time till the next interest rate step and the probability that interest rates will fall next time the central bank decides to take action. Some of the propositions we derive in this respect lend themselves to empirical testing. For instance, the model predicts that the size of the interest rate step will be increasing in the cost of changing the interest rate and the volatility of the demand shock and decreasing in the slope of the Lucas supply curve. Similarly, the average duration between two consecutive steps will be decreasing in the interest rate sensitivity of aggregate demand and the volatility of the demand shock and increasing in the slope of the Lucas supply function and the cost of changing the interest rate. Finally, we examine the effect of these 'menu' costs on inflationary expectations. We show that the economy will suffer from an inflationary bias if the cost of raising the interest rate exceeds the cost of lowering. This result is interesting since it shows that an inflationary bias can arise even if the central bank does not try to create surprise inflation in pursuit of various real objectives.

In line with the literature on monetary policy rules, our model clearly distinguishes between the interest rate as the control variable and the rate of inflation as a state variable. However, it differs from most other models in assuming that inflation is instantaneously and perfectly controllable, i.e. it abstracts from lags in the transmission process. Nevertheless, in these models the conditional inflation forecast, which serves as the intermediate target of monetary policy, *can* be perfectly and instantaneously controlled. Hence, in our view the rate of inflation in our model is best viewed as the conditional inflation forecast when considering the implications of the model for the real world. In this sense the model provides an explanation for the existence of bands for the intermediate target of monetary policy even if this intermediate target itself is perfectly controllable.

Appendix A: Derivation of the optimal band of inaction

Using the fact that $e^{-\delta dt} \approx 1 - \delta dt$ we can rewrite equation **(3.13)** as follows:

$$\begin{aligned} L(x) &= x^2 dt + (1-\delta dt)\{L(x) + E(L(x+dx) - L(x))\} \\ &= L(x) + x^2 dt - \delta L(x) dt + E(dL(x)) - \delta dt E(dL(x)) \end{aligned} \quad \textbf{(A.1)}$$

Since $dx = \theta\sigma dw$, by Ito's Lemma it holds that:

$$E(dL(x)) = \frac{1}{2}\theta^2\sigma^2 L''(x)dt \quad \textbf{(A.2)}$$

Substituting this equation in **(A.1)**, ignoring terms which are small relative to dt and subsequently dividing by dt will yield the second-order differential equation **(3.13)** in the main text. The solution to this equation consists of the sum of a particular solution ($L_p(x)$) and the general solution of the homogeneous part:

$$L(x) = L_p(x) + Ae^{q_1 x} + Be^{q_2 x} \quad \textbf{(A.3)}$$

Here A and B are constants to be determined and q_1 and q_2 are the roots of the characteristic equation of the homogeneous part.

Since the forcing term is quadratic in x, we try the following particular solution:

$$L_p(x) = d_0 x^2 + d_1 x + d_2 \quad \textbf{(A.4)}$$

Plugging the resulting expressions for L'' (x) and L(x) in equation **(3.13)** and subsequently equating coefficients across equations **(3.13)** and **(A.4)** yields: $d_0 = 1/\delta$, $d_1 = 0$ and $d_2 = (\theta^2\sigma^2)/\delta^2$. As in Dixit (1993) the resulting particular solution can be thought of as the present value of the intertemporal loss function under the condition that control is never exercised[48]. Consequently, the effect of barriers will be fully captured by the complementary function. To find this function we solve the characteristic equation of the homogeneous part to obtain the following expression for the characteristic roots:

$$\frac{1}{2}\theta^2\sigma^2 q^2 - \delta = 0 \qquad \Leftrightarrow \qquad q_{1,2} = \pm\frac{\sqrt{2\delta}}{\sigma\theta} \tag{A.5}$$

Next, regarding equation **(A.3)** we note that the threshold level (b) will only affect the constants A and B, since the band is symmetric we therefore must have: A=B. Defining $\gamma = |q|$ and substituting the particular solution and equation **(A.5)** into **(A.3)** yields equation **(3.14)** in the main text.

Finally, we can solve for b using the analytical approximation developed by Dixit (1991). Dividing the VMC-condition **(3.15)** by the SPC-condition **(3.16)** yields:

$$\frac{\left(e^{\gamma b} + e^{-\gamma b} - 2\right)}{\gamma b\left(e^{\gamma b} - e^{-\gamma b}\right)} = \frac{1}{2}\left[1 - \frac{C\delta}{b^2}\right] \tag{A.6}$$

Provided γb is sufficiently small in a manner to be explained, the LHS can be approximated by a fourth-order Taylor expansion around $\gamma b = 0$:

48 This can easily be seen by plugging the particular solution into equation **(3.13)**. The reason for this result is that the Bellman equation **(3.12)** is valid for $x \in (-b,b)$ which is the region in which control is never exercised.

$$\frac{1+\gamma b+\frac{1}{2}(\gamma b)^2+\frac{1}{6}(\gamma b)^3+\frac{1}{24}(\gamma b)^4+(1-\gamma b+\frac{1}{2}(\gamma b)^2-\frac{1}{6}(\gamma b)^3+\frac{1}{24}(\gamma b)^4)-2}{\gamma b[1+\gamma b+\frac{1}{2}(\gamma b)^2+\frac{1}{6}(\gamma b)^3+\frac{1}{24}(\gamma b)^4-(1-\gamma b+\frac{1}{2}(\gamma b)^2-\frac{1}{6}(\gamma b)^3+\frac{1}{24}(\gamma b)^4)]} =$$

$$\frac{\gamma^2 b^2+\frac{1}{12}\gamma^4 b^4}{\gamma b\left(2\gamma b+\frac{1}{3}\gamma^3 b^3\right)} = \frac{1}{2}\left(\frac{1+\frac{1}{12}\gamma^2 b^2}{1+\frac{1}{6}\gamma^2 b^2}\right) = \tag{A.7}$$

$$\frac{1}{2}\left(1+\frac{1}{12}\gamma^2 b^2\right)\left(1-\frac{1}{6}\gamma^2 b^2\right) = \frac{1}{2}\left(1-\frac{1}{12}\gamma^2 b^2\right)$$

Equating the outcome of this approximation to the RHS of equation **(A.6)** and solving for b yields equation **(3.17)** in the main text.

Finally, we will examine under which conditions γb will be sufficiently small. Take the following parameter values: $\delta = 0.05$, $\beta = 2$, $\alpha = 1$, $C = 0.01$ and $\sigma = 0.1$. Plugging these values into the expressions obtained for γ and b and subsequently computing the product yields: $\gamma b \approx 0.5$. Since higher-order terms in the expansion of the LHS of **(A.6)** involve terms like $(\gamma b)^5/120$ and smaller we can conclude that the approximation is quite robust.

Appendix B: Derivation of the Probability of an Interest Rate Decrease and the Expected Time Period till the next Interest Rate Step

Follwing Dixit (1993), let Q(x) denote that probability that x will first hit the lower barrier. Furthermore assume that x is regulated within the band (-a,b) where a,b > 0. For any x within this band it will hold that:

$$Q(x) = \frac{1}{2}Q(x-dx)+\frac{1}{2}Q(x+dx) \tag{B.1}$$

Rewriting this equation and dividing by $(dx)^2$ yields the following:

$$0 = \frac{[Q(x+dx) - Q(x)] - [Q(x) - Q(x-dx)]}{(dx)^2} \quad \textbf{(B.2)}$$

Taking the limit of the RHS of this equation as dx→0 we have: $Q''(x) = 0$. Therefore the general solution for Q(x) will be:

$$Q(x) = Fx + H \quad \textbf{(B.3)}$$

where F and H are constants to be determined by examining Q(x) at the boundaries. This yields:

$$\begin{array}{lll} Q(-a) = 1 & \Leftrightarrow & -aF + H = 1 \\ Q(b) \ = 0 & \Leftrightarrow & bF + H = 0 \end{array} \quad \textbf{(B.4)}$$

Solving for F and H we find:

$$Q(x) = \frac{b-x}{b+a} \quad \textbf{(B.5)}$$

Next, let T(x) denote the expected time period till the next interest rate step. For simplicity we assume that x is regulated within the symmetric band (-b,b). For any x which is strictly in the interior of this band we have:

$$T(x) = dt + \frac{1}{2}T(x+dx) + \frac{1}{2}T(x-dx) \quad \textbf{(B.6)}$$

Rewriting this and dividing both sides by $(dx)^2$ we have:

$$\frac{-2dt}{(dx)^2} = \frac{[T(x+dx) - T(x)] - [T(x) - T(x-dx)]}{(dx)^2} \quad \textbf{(B.7)}$$

From equation **(3.11)** it follows that $(dx)^2 = \theta^2\sigma^2 dt$. Using this on the LHS of equation **(B.7)** and subsequently taking the limit for dx→0 on the RHS yields:

$$T''(x) = \frac{-2}{\theta^2\sigma^2} \qquad \textbf{(B.8)}$$

Since the RHS of this equation is a constant we try a solution of the form:

$$T(x) = Lx^2 + Mx + N \qquad \textbf{(B.9)}$$

Using equation **(B.8)** it can be seen that $L = -1/(\theta^2\sigma^2)$. Next, from the condition that T(-b)=T(b)=0 we can establish: M=0 and $N = b^2/(\theta^2\sigma^2)$. Plugging these values into **(B.9)** and using the expression obtained for b in equation **(3.17)** yields equation **(3.19)** in the main text.

Appendix C: Derivation of the Long-Run Stationary Distribution for x_t

Consider the variable x_t which follows the Brownian motion described in equation **(3.11)** and which is regulated within the band (-a,b) where a,b > 0. For any $x_t \in (-a,b)$ let:

$$\begin{aligned} x_{t+dt} &= x_t + dx \quad \text{with prob } \frac{1}{2} \\ &\quad\; x_t - dx \quad \text{with prob } \frac{1}{2} \end{aligned} \qquad \textbf{(C.1)}$$

From this, the stationary probability density function ($\phi(x)$) must satisfy:

$$\phi(x) = \frac{1}{2}\phi(x-dx) + \frac{1}{2}\phi(x+dx) \qquad \textbf{(C.2)}$$

Rewriting this and dividing by $(dx)^2$ yields:

$$0 = \frac{[\phi(x+dx) - \phi(x)] - [\phi(x) - \phi(x-dx)]}{(dx)^2} \quad \textbf{(C.3)}$$

Taking the limit for $dx \to 0$ on the RHS of **(C.3)**, it follows that $\phi''(x) = 0$. Consequently, the general solution for $\phi(x)$ will be:

$$\phi(x) = Fx + G \quad \textbf{(C.4)}$$

where F and G are constants which can be determined by examining the behaviour of $\phi(x)$ at the boundaries and the resetting point. First for $x_t = -a + dx$ it will hold that:

$$\begin{aligned} x_{t+dt} &= -a + 2dx \qquad with \quad prob \quad \frac{1}{2} \\ &= 0 \qquad\qquad\quad with \quad prob \quad \frac{1}{2} \end{aligned} \quad \textbf{(C.5)}$$

From this we can conclude:

$$\begin{aligned} &\phi(a) = 0 \\ &\phi(-a + 2dx) = 2\phi(-a + dx) \end{aligned} \quad \textbf{(C.6)}$$

Furthermore, since $\phi(-a+2dx)$ will satisfy equation **(C.1)**, it can easily be shown that for $n \geq 1$ and for $-a < x < 0$, it holds that:

$$\phi(-a + ndx) = n\phi(-a + dx) \quad \textbf{(C.7)}$$

Similarly, for the upper boundary b it can be shown that:

$$\begin{aligned} &\phi(b) = 0 \\ &\phi(b - mdx) = m\phi(b - dx) \end{aligned} \quad \textbf{(C.8)}$$

where the second equation holds for for $m \geq 1$ and $0 < x < b$. From equations **(C.4)**, **(C.7)** and **(C.8)** we can see that $\phi(x)$ will be linearly increasing in x for $x \in [-a,0)$ and linearly decreasing in x for $x \in (0,b]$. It remains to examine $\phi(0)$ for which it holds that:

$$\phi(0) = \frac{1}{2}\phi(0-dx) + \frac{1}{2}\phi(0+dx) + \frac{1}{2}\phi(-a+dx) + \frac{1}{2}\phi(b-dx) \qquad \textbf{(C.9)}$$

Rearranging and taking limits (see also Bertola and Caballero (1990)) we can write:

$$\begin{aligned} &\lim_{dx\uparrow 0}\frac{\phi(0)-\phi(0-dx)}{dx} + \lim_{dx\uparrow 0}\frac{\phi(b)-\phi(b-dx)}{dx} = \\ &\lim_{dx\downarrow 0}\frac{\phi(0+dx)-\phi(0)}{dx} + \lim_{dx\downarrow 0}\frac{\phi(-a+dx)-\phi(-a)}{dx} \end{aligned} \qquad \textbf{(C.10)}$$

From this equation it follows that while $\phi(x)$ is continuous at x=0 (this is ensured by **(C.9)**), it is not differentiable at this point since the RHS and the LHS derivatives at x=0 have opposite signs.

Consequently, the Brownian motion process for x_t subject to barriers –a and b will give rise to a triangular steady state probability density function the support of which is determined by the control thresholds (see Figure 3.2).

Finally, $\phi(0)$ can be determined by the requirement that $\int_{-a}^{b} \phi(x)\, dx = 1$. From Figure 3.2 it can be seen that this boils down to the the condition that $\frac{1}{2}(a+b)\,\phi(0) = 1$. Using this, we obtain equation **(3.20)** in the main text.

Appendix D: Proof of Propositions

Proposition 3.2:

The sign of the partial derivatives of $|\, i_\tau - i_0 \,|$ with respect to C, β and σ can be unambiguously inferred from equation **(3.18)**. As for the parameter α we can compute:

$$\frac{\partial\,|\,i_\tau - i_0\,|}{\partial\alpha} = \frac{(3\alpha-2\beta)(6C\sigma^2)^{\frac{1}{4}}}{2\alpha^2(\beta-\alpha)^{\frac{3}{2}}} \begin{matrix} > \\ < \end{matrix} 0 \quad if \quad \frac{\beta}{\alpha} \begin{matrix} < \\ > \end{matrix} \frac{3}{2} \tag{D.1}$$

Proposition 3.3:

From equation **(3.19)** in the main text we can compute:

$$\frac{\partial T(x)}{\partial c} = \frac{\sqrt{\frac{3}{2}}(\beta-\alpha)}{\sigma\sqrt{C}} > 0$$

$$\frac{\partial T(x)}{\partial \beta} = -\frac{\partial T(x)}{\partial \alpha} = \frac{\sigma\sqrt{6C} - 2(\beta-\alpha)x^2}{\sigma^2} \tag{D.2}$$

$$\frac{\partial T(x)}{\partial \sigma} = -\frac{(\beta-\alpha)\left[\sigma\sqrt{6C} - 2(\beta-\alpha)x^2\right]}{\sigma^3}$$

From this it can be seen that for T(x) to be increasing in β and decreasing in α and σ we need to have:

$$\sigma\sqrt{6C} > 2(\beta-\alpha)x^2 \quad \Leftrightarrow \quad |x| < \left(\frac{\sigma\sqrt{6C}}{2(\beta-\alpha)}\right)^{\frac{1}{2}} \quad \Leftrightarrow \quad |\,x\,| < \frac{b}{\sqrt{2}} \tag{D.3}$$

Proposition 3.4:

From equation **(3.20)** in the main text we can compute (note that $x \in (-a, b)$) :

$$\frac{\partial Q(x)}{\partial b} = \frac{a+x}{(a+b)^2} > 0 \quad ; \quad \frac{\partial Q(x)}{\partial a} = \frac{x-b}{(a+b)^2} < 0 \tag{D.4}$$

a and b (together with two constants of integration) are determined by the two Value

Matching Conditions: $L(-a)-L(0) = C_l$, $L(b)-L(0) = C_h$ and the two Smooth Pasting Conditions: $L'(-a) = L'(b) = 0$. From these it can easily be shown that $\partial b/\partial C_h > 0$ and that $\partial a/\partial C_l > 0$ (see Dixit (1993)). Consequently we have: $\partial Q(x)/\partial C_h > 0$ and $\partial Q(x)/\partial C_l < 0$.

Proposition 3.5:

To prove that π^e is increasing in C_h and decreasing in C_l it is sufficient to prove that this holds for E(x). From equation **(3.21)** we have:

$$\frac{\partial E(x)}{\partial b} = \frac{1}{3} \qquad ; \qquad \frac{\partial E(x)}{\partial a} = -\frac{1}{3}$$

Using the results obtained in the proof of Proposition 3.4 then yields the proof of this proposition.

Appendix E: Figures 3.1 and 3.2

Figure 3.1: The Optimal Band Width

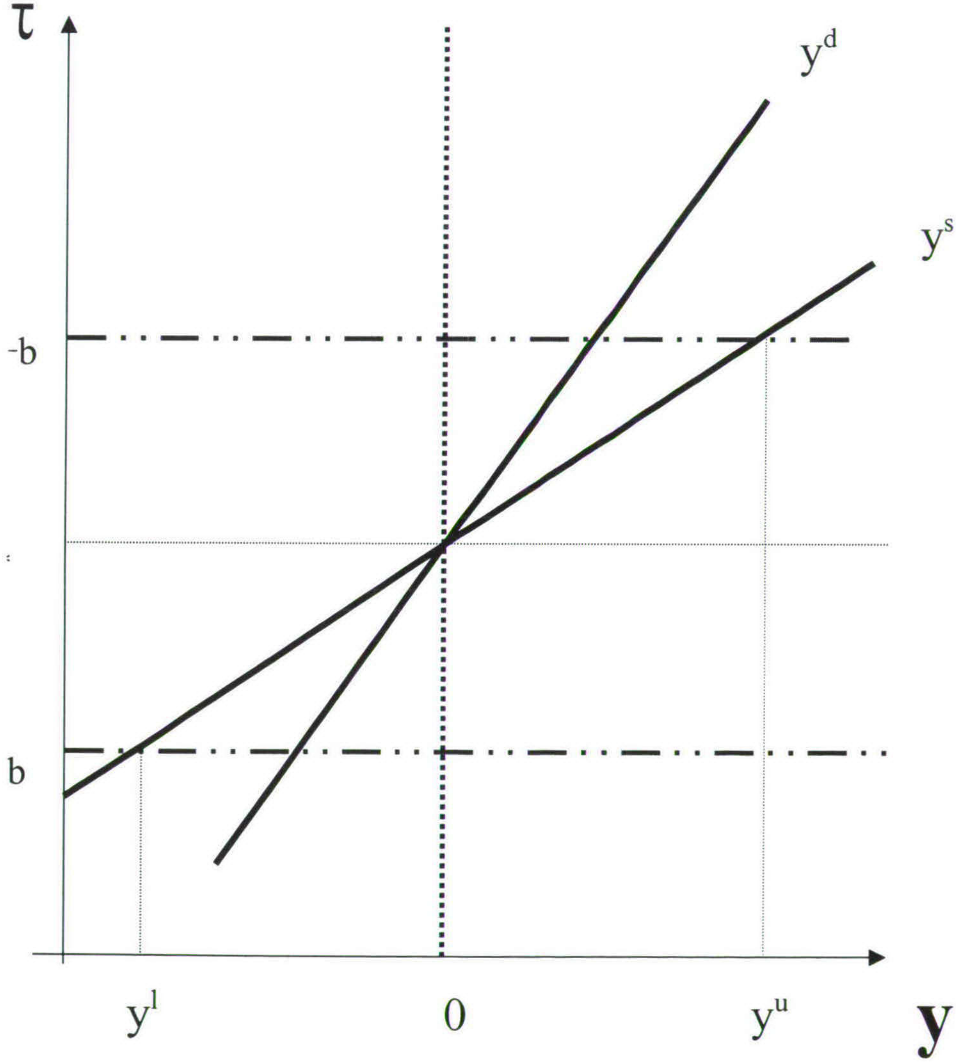
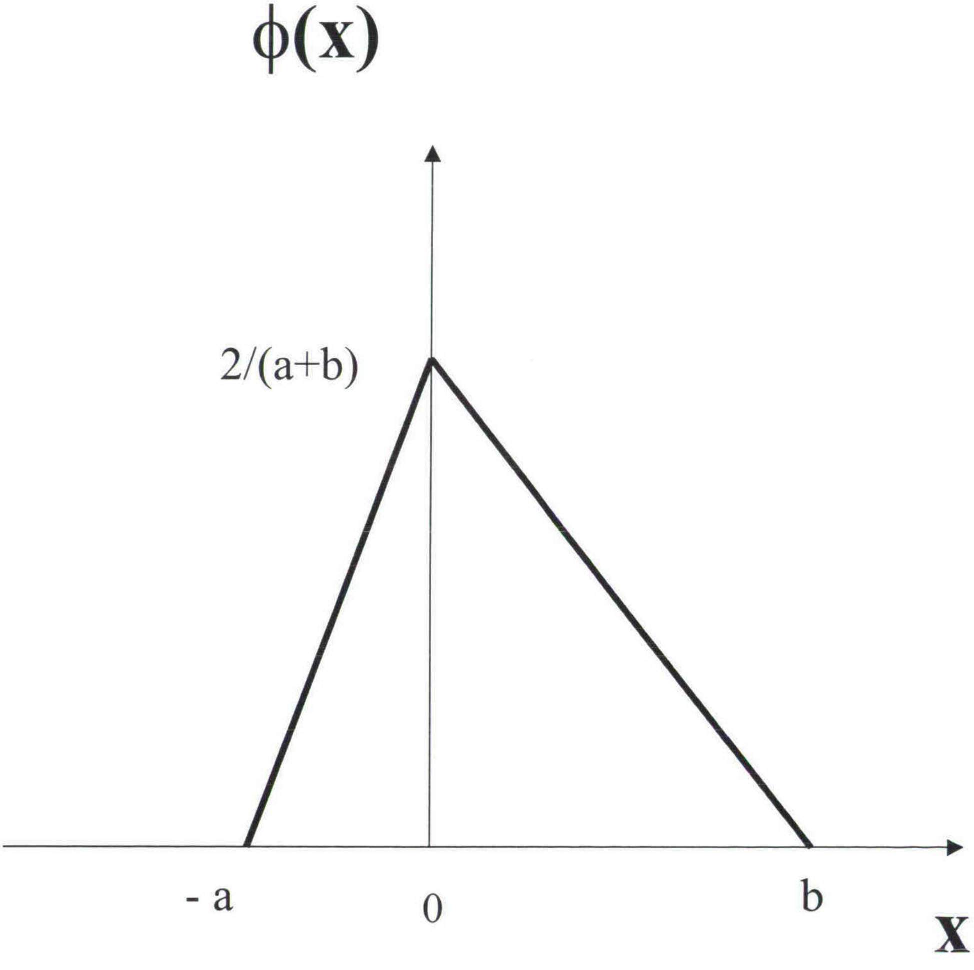


Figure 3.2: Probability Density Function for the Inflation Gap



Part II : The Optimal Relative Weight on Output Stabilisation in an Inflation Targeting Regime

Although central bankers are concerned with maximising social welfare this does not necessarily mean that their preferences should perfectly reflect society's preferences concerning inflation and output stabilisation. One of the first contributions which formalised this important point was made by Rogoff (1985). He showed that if the monetary policymaker is tempted to push output above the natural rate systematically, society will gain if it appoints a central banker whose relative weight on output stabilisation is lower than society's relative concern for output stabilisation. The reason is that such a *conservative* central banker will generate a lower average rate of inflation (i.e. the credibility of monetary policy will improve). However, this will come at the cost of a diminished flexibility to respond to supply shocks in accordance with social preferences.

This result has led other researchers to search for mechanisms which would lead to an improved credibility-flexibility trade-off. Among these are the injection of political uncertainty (i.e. uncertainty about the central bank's relative concern for inflation stabilisation see e.g. Eijffinger, Hoeberichts and Schaling (2000)), the design of an optimal central bank contract (Walsh (1995)) or the assignment of an explicit inflation target (Svensson (1997a)). The latter two solutions can in principle achieve the desirable situation in which the inflationary bias disappears completely and in which output shocks are stabilised in accordance with society's preferences.

By contrast, the literature which deals with the implementation of inflation targeting (e.g. Svensson (1997b) and many papers based on this model) has paid relatively little attention to this parameter in the central banker's loss function. The reason for this may be that this literature typically assumes that the central banker does not seek to drive output above the natural rate systematically. This assumption follows the criticism raised by some prominent economists (e.g. McCallum (1995) and Blinder (1998)) that independent central bankers will realise that an output target which exceeds the natural rate will induce an inflationary bias and will therefore choose the long run natural rate as their target rate for output. At first sight, the absence of an inflationary bias in monetary policy then seems to obviate the need for a central banker who is more inflation averse than society.

An important exception in this literature is a recent paper by Clarida, Gali and Gertler (1999) who show that society *will* gain from appointing a conservative central banker even if the output target is equal to the natural rate if price setting depends on expected *future* output gaps (i.e. if the Phillipscurve is of the New-Keynesian variety in which inflation in period t depends on the expectation of inflation in period t+1 conditional on the information set in period t). The reason is that by internalising the effect of current actions on future inflation, the central bank will face an improved inflation-output trade-off.

The purpose of Chapters 4 and 5 is to investigate to what extent this result is robust to two other widely used specifications of the economy's aggregate supply relationship. It may be important to check this since there is no general agreement about the appropriate nature of this relationship in the economics profession. Because of this Cukierman (2000, p. 14) states that "*...It is likely that he* (i.e. the central banker) *is going to intuitively assign some non negative weight to each of the models...* ".

In Chapter 4 we will determine the optimal degree of conservatism under the assumption that price setting is purely backward looking. In this sticky price model output will be determined by aggregated demand. Given the level of potential output, a change in aggregate demand will affect the output gap (defined as the difference between actual and potential output) and subsequently the rate of inflation. We present a simplified version of the well-known inflation forecast targeting model first introduced by Svensson (1997b) and find that in this setting it is indeed optimal to appoint a conservative central banker. The reason is that a central banker acting under discretion will not take the effect of current actions on future inflation into account. Subsequently, we extend the model with uncertainty about the potential level of output to see whether or not this will affect the optimal degree of conservatism.

In Chapter 5 we analyse the traditional linear expectations-augmented Phillipscurve (which can be seen as the inverse of the Lucas supply function). In this model, actual output will in principle be determined by a combination of supply and demand factors. However, the latter can only influence actual output to the extent that demand factors are not incorporated into inflationary expectations. In other words, whether or not short-run neutrality holds in this model depends crucially on whether or not the central bank has private information. Hence, we will analyse the optimal degree of conservatism in a model where the central bank has private information about the cost-push shock but the public can partially predict its realisation and the case where no surprise inflation is possible. Next, we extend the model by departing from the certainty equivalent framework in assuming that the Phillipscurve is convex. The implications of a non-linear Phillipscurve for monetary policy in general have been analysed by Bean (2000) in the context of loss function where output enters in a linear fashion and by Schaling (1998) for the case of strict inflation targeting. Here we use the familiar quadratic loss function featuring the output gap and inflation and find that some degree of output stabilisation will improve social welfare *even* if the central banker cannot generate a surprise inflation.

Chapter 4: Inflation Targeting and The Optimal Degree of Conservatism with and without Uncertainty about Potential Output

4.1: Introduction

Since the widespread adoption of inflation targeting many researchers have addressed the question how best to implement such a regime. One of the seminal contributions in this respect has been Svensson (1997b) who argues that inflation targeting can be seen as a situation in which the government assigns a particular loss function to the central bank. Basically, the central bank's task is to minimise a weighted sum of the variability of inflation around an explicitly assigned target and the variability of output around its potential. The latter assumption is a marked deviation from the literature dealing with the inflationary bias problem in which society is burdened with a suboptimally high rate of inflation due to the fact that the monetary policymaker systematically tries to push output above potential. However, as argued by many authors (e.g. Blinder (1998)), while an ambitious output target may represent the preferences of politically subservient policymakers, central bankers who have been granted instrument independence *do* realise the danger and the futility of this.

Rogoff (1985) suggested that the decrease in social welfare stemming from an ambitious output target can be partly compensated by changing another aspect of the central banker's loss function. He showed that a policymaker who is conservative in the sense that she is more inflation averse than society will generate a lower average rate of inflation. However, this will come at the cost of a suboptimally low degree of output stabilisation. By contrast, the optimal relative weight on output stabilisation has received little attention in the literature dealing with the implementation of inflation targeting. At first sight it may seem that the absence of an inflationary bias obviates the need for appointing a conservative central banker. Nevertheless, in a recent contribution Clarida, Gali and Gertler (1999) showed that social welfare will be higher under a conservative policymaker when she is faced with a New-Keynesian

Phillipscurve. The reason is that in this case inflation will be determined by expected future values of the output gap. The difference between policymaking under commitment and under discretion is that in the latter case the central banker will not take the effect of current actions on future inflation-output trade-offs into account. For this reason social welfare will be higher if the policymaker can commit herself. However, in the absence of a credible commitment mechanism the commitment solution can be replicated by appointing a conservative central banker. The purpose of this chapter is to see whether this result also holds under the backwardlooking (or accelerationist) Phillipscurve which, despite its lack of theoretical foundations, is still frequently used in both the academic literature and actual policymaking (see e.g. Cukierman (2000)). Under this specification of the economy's aggregate supply relationship the rate of inflation generated today will also affect the inflation-output trade-off in the next period. Our main finding is that it will therefore also be socially optimal to appoint a central banker who is more inflation averse than society in this case.

Subsequently, we extend the model by assuming that the central banker cannot observe potential output in real time but she regularly receives updates on past values of potential output. According Rudebusch (2000) this corresponds to the situation actually faced by real world policymakers. This begs the question whether or not the presence of this kind of uncertainty should make the central banker even more conservative. Clearly, uncertainty about potential output will increase both the variability of inflation and the output gap. To the extent that it alters these variabilities to a different degree it may also affect the socially optimal relative weight on output stabilisation. However, we find that under the assumptions made in this chapter (most notably the fact that the Phillipscurve is of the accelerationist variety and the fact the central banker uses past inflation rates to optimally filter out information about the current level of potential output), the presence of uncertainty about potential output will not change the optimal degree of conservatism relative to the case where potential output is fixed and known to the policymaker.

4.2: The Optimal Degree of Conservatism when Potential Output is Fixed

In this section we present a simplified version of Svensson's inflation forecast targeting framework (see Svensson (1997b)). The Phillipscurve in this case reads as follows:

$$\pi_t \;=\; \pi_{t-1} \;+\; \alpha(y_t - y_t^*) \;+\; v_t \tag{4.1}$$

Inflation in the current period (π_t) depends positively on inflation in the previous period, the output gap and a cost-push shock (v_t). The output gap is defined as the difference between actual output (y_t) and potential output (y_t^*). In this section we assume that potential output is fixed and without loss of generality we normalise it to zero. The cost-push shock is an i.i.d. normal with zero mean and variance σ_v^2. Since this is a sticky price model, actual output will be determined by aggregate demand which, in turn, is given by the following relationship:

$$y_t \;=\; -(i_t - E_{t-1}\pi_{t+1}) \;+\; \varepsilon_t \tag{4.2}$$

As usual demand depends negatively on the real interest rate ($i_t - E_{t-1}\pi_{t+1}$) where the expected rate of inflation is conditional on the public's and the central bank's information set at the end of period t-1. Next, y_t is also subjected to a demand shock (ε_t) which follows an i.i.d. normal distribution with mean zero and variance equal to σ_ε^2. We do not explicitly take account of lags in the transmission mechanism but instead approximate the effect of them by assuming that the central bank and the public cannot observe the current realisations of the supply shock (v_t) and the demand shock (ε_t).

The objective of the central bank is so stabilise inflation around the assigned target (which without loss of generality is normalised to zero) and to stabilise ouput around its potential:

$$L_t \;=\; 2(1-\delta)E_t\left[\sum_{j=t}^{\infty}\delta^{j-t}\left[\frac{1}{2}\pi_j^2 \;+\; \frac{\lambda}{2}(y_j - y_j^*)^2\right]\right] \tag{4.3}$$

We restrict ourselves to the case of pure discretion in which the central bank is free to set the interest rate at whatever it considers to be the optimal level in each and every period. Moreover, there is nothing agents can learn from past central bank actions about either the preferences of the central bank and/or its knowledge of the economy.[49] This means that there is no link between periods because of which the central banks problem in period t boils down to minimising the period t loss function subject to the Phillips-curve constraint. From the aggregate demand equation **(4.2)** it can be seen that $E_{t-1}y_t$ can be regarded as an indirect control variable for the central bank. Minimising the period t loss function with respect to y_t then yields the following first-order condition:

$$E_{t-1}y_t = -\frac{\alpha}{\lambda}E_{t-1}\pi_t \qquad \textbf{(4.4)}$$

This first-order condition is exactly the same as the one obtained by Clarida, Gali and Gertler (1999) and shows that the central bank essentially pursues a 'leaning against the wind' policy by contracting the output gap whenever the conditional inflation forecast is above the target and vice versa. To obtain closed form solutions for inflation and output we take expectations conditional on the information set at the end of period t-1 across equation **(4.1).** Substituting the resulting expression for $E_{t-1}\pi_t$ into **(4.4)** and realising that $y_t = E_{t-1}y_t + \varepsilon_t$ we obtain the following for the output gap in period t:

$$y_t - y_t^* = -\frac{\alpha}{\lambda+\alpha^2}\pi_{t-1} + \varepsilon_t \qquad \textbf{(4.5)}$$

Intuitively, the central bank will contract output whenever the inherited rate of inflation exceeds the target while the degree of activism with which it will do so is decreasing in the relative weight on output stabilisation (λ).

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[49] This would be true even if the central bank had an information advantage over the public in the sense that it knows the realisation of the supply shock when setting monetary policy. In that case past central bank actions or macroeconomic outcomes would not help the private sector to form a better estimate of the innovation to the supply shock.

Plugging equation **(4.5)** into equation **(4.1)** yields the equilibrium rate of inflation in period t:

$$\pi_t = \frac{\lambda}{(\lambda+\alpha^2)}\pi_{t-1} + v_t + \alpha\varepsilon_t \tag{4.6}$$

Optimal monetary policy will cause the equilibrium rate of inflation to be serially correlated and stationary where the resulting *endogenous* persistence parameter for inflation will be increasing in the relative weight on output stabilisation and decreasing in the slope of the Phillipscurve. Hence, as in Svensson (1997b) the central bank's relative weight on output stabilisation governs the *speed* with which inflation is brought back in line with the inflation target because of which it is an important determinant of the trade-off between inflation and output variability.

Next, using equations **(4.2)** and **(4.6)** we can derive the period t optimal nominal interest rate:

$$i_t = \frac{\alpha(\lambda+\alpha^2)+\lambda^2}{(\lambda+\alpha^2)^2}\pi_{t-1} \tag{4.7}$$

Interestingly, the effect of the central bank's relative weight on output stabilisation on the degree of activism with which it responds to the inherited rate of inflation (π_{t-1}) turns out to be ambiguous. On the one hand, if π_{t-1} is positive, an increase in λ will cause an increase in the expected rate of inflation (which from equation **(4.6)** can be seen to be equal to $(\lambda^2\pi_{t-1})/(\lambda+\alpha^2)^2$). All else equal, this will cause the nominal interest rate to go up as well. On the other hand, an increased concern for output stabilisation will cause the *real* interest rate ($r_t = (\alpha\pi_{t-1})/(\lambda+\alpha^2)$) to increase less in response to a rise in π_{t-1}. In the Svensson (1997b) model, the first effect is absent since the expected rate of inflation is completely predetermined because of a two period lag between a change in the interest rate and its effect on inflation.

We assume that the choice of the optimal degree of output stabilisation is a 'once-and-for-all' decision. In other words, it is determined in the institutional design phase and subsequently announced to the public. Hence, to determine the optimal value of λ we need the unconditional expected value of society's loss function which is assumed to be the same as the central bank's loss function expect for the fact that society's relative weight on output stabilisation (ξ) may differ:

$$E(L) = Var(\pi) + (E(\pi))^2 + \xi\left[Var(y - y^*) + (E(y - y^*))^2\right] \qquad \textbf{(4.8)}$$

From equations **(4.5)** and **(4.6)** we can see that E(π) = E(y) = 0. Furthermore, from these equations, the unconditional variances of inflation and the output gap can be easily computed:

$$Var(\pi) = \frac{(\lambda + \alpha^2)^2}{(2\lambda + \alpha^2)}\left[\sigma_\varepsilon^2 + \frac{1}{\alpha^2}\sigma_v^2\right]$$

$$Var(y - y^*) = \left(1 + \frac{\alpha^2}{(2\lambda + \alpha^2)}\right)\left[\sigma_\varepsilon^2 + \frac{1}{\alpha^2}\sigma_v^2\right] - \frac{\sigma_v^2}{\alpha^2} \qquad \textbf{(4.9)}$$

We can now obtain the optimal degree of output stabilisation (λ*) by inserting equation **(4.9)** into the social loss function **(4.8)** and minimising the resulting expression with respect to λ:[50]

$$\lambda^* = \frac{-\alpha^2 + \alpha\sqrt{\alpha^2 + 4\xi}}{2} \qquad \textbf{(4.10)}$$

[50] This minimisation will yield two solutions for λ*, one of which is negative and one of which is positive. Obviously, the latter one is the relevant solution here.

Proposition 4.1:

If the central bank is faced with an accelerationist Phillipscurve and if potential output is fixed and known, it will be optimal to appoint a conservative central banker ($0 < \lambda^* < \xi$) even if the central banker's output target is equal to potential output. Moreover, the optimal degree of conservatism in that case will decrease (i.e. λ^* will increase) if:

1. the slope of the Phillipscurve (α) increases
2. society's relative weight on output stabilisation (ξ) increases.

Proof: see Appendix A

The intuition is that a central bank acting under discretion will not internalise the effect of its current decision on next period's optimisation problem. Nevertheless, the accelerationist Phillipscurve *does* introduce a link between periods since the rate of inflation in period t *will* affect the inflation-output trade-off in period t+1. If the central bank could commit, it would explicitly take this into account when setting monetary policy in period t. This recognition would lead to an improved trade-off between inflation and output variability. However, as in Clarida, Gali and Gertler (1999), if commitment is not feasible social welfare can be improved by appointing a conservative central banker.

As for the parameters affecting the optimal degree of conservatism, an increase in the slope of the Phillipscurve (α) corresponds to a decrease in the endogenously induced persistence of inflation as can be seen from equation **(4.6).**[51] As a result, the variability of inflation caused by the variance of supply shocks (σ_v^2) will decrease and there will be more room for the central bank to engage in output stabilisation. Furthermore, an increase in society's optimal relative weight on output stabilisation (ξ) will spill over into a lower degree of optimal central bank conservatism.

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[51] It should be noted that this endogenous persistence is induced by the fact that the central bank cares about output stabilisation ($\lambda > 0$). In the case of strict inflation targeting, π_t would not depend on π_{t-1}.

4.3: Optimal Monetary Policy with Uncertainty about Potential Output

In this section we assume that the central bank faces uncertainty about supply side developments. The possible importance of such uncertainty is highlighted by Bullard and Schaling (2000) who analyse the implications for monetary policy when the cost-push shockis subjected to a Markov regime switching process in the context of strict inflation targeting. Here we will assume the central bank faces uncertainty about the level of potential output (y^*) and will try to assess the implications of this uncertainty for the optimal relative weight on output stabilisation. In particular, we assume that potential output is subject to the following serially correlated process:

$$y_t^* = \rho y_{t-1}^* + \tau_t \tag{4.11}$$

This equation can be thought of as a situation in which potential output fluctuates randomly along some long-term trend where the latter is normalised to zero in this model. We also assume that these fluctuations are persistent and stationary (i.e. $0< \rho< 1$). The innovation to potential output (τ_{t-1}) follows an i.i.d. normal distribution with mean zero and variance σ_τ^2.

Because of the persistence in potential output, the central bank can use its past realisations to estimate the current level. In particular, we assume the central bank has a perfect observation on potential output in period t–2 (y_{t-2}^*). As observed Rudebusch (2000) this corresponds to a situation actually faced by many central banks in which data on potential output are hardly ever available in real time and in which current *estimates* of potential output are frequently revised at a later date. If in addition we assume the central bank knows the realisations of inflation and demand in the previous period (π_{t-1} and y_{t-1}, respectively), its estimate of the current level of potential output will be (see Appendix B):

$$E_{t-1}y_t^* = \rho^2 y_{t-2}^* - \frac{\rho\theta}{\alpha}\Omega_{t-1} \qquad \textit{where} \qquad \theta = \frac{\alpha^2\sigma_\tau^2}{\alpha^2\sigma_\tau^2 + \sigma_v^2}$$

$$\textit{and} \qquad \Omega_{t-1} = -\alpha\tau_{t-1} + v_{t-1} = \pi_{t-1} - \pi_{t-2} - \alpha y_{t-1} - \rho^2 y_{t-2}^* \tag{4.13}$$

Basically, an observation on π_{t-1} amounts to an observation on what could be termed last period's *inflation residual* (Ω_{t-1}). This is simply the part of last period's inflation rate which cannot be accounted for by the realisations of variables which are known to the central bank at the end of period t-1 (i.e. by π_{t-2} , y_{t-1} and y_{t-2}^*). Since the central bank knows that the realisation of Ω_{t-1} is caused by a linear combination of last period's innovation to potential output (τ_{t-1}) and last period's cost-push shock (ν_{t-1}), it will use its knowledge of the probability distributions of these shocks to optimally filter out information on τ_{t-1}.[52]

Minimising the central bank's loss function **(4.3)** with respect to y_t yields the following first order condition:

$$E_{t-1}y_t \quad = \quad -\frac{\alpha}{\lambda}E_{t-1}\pi_t \quad + \quad \rho^2 y^*_{t-2} \quad -\frac{\rho\theta}{\alpha}\Omega_{t-1} \qquad \textbf{(4.14)}$$

Hence, the extent to which the central bank will contract or expand output also depends on its estimate of supply side developments. First of all, the central bank's update of potential output in period t-2 will spill over into the central bank's optimal output target today because of the persistence in the process driving potential output. Next, all else equal, a positive realisation of last period's inflation residual (Ω_{t-1}) will induce the central bank to contract demand since part of it will be attributed to adverse supply side developments (i.e. a negative realisation of τ_{t-1}).

Taking rational expectations based on the central bank's information set at the end of period t-1 across equation **(4.1)**, inserting the result into equation **(4.14)** and realising that $y_t - y_t^* = E_{t-1}y_t + \varepsilon_t - \rho^2 y_{t-1}^* - \rho\tau_{t-1} - \tau_t$, we obtain the following expression for the output gap in period t:

$$y_t - y_t^* \quad = \quad -\frac{\alpha}{\lambda+\alpha^2}\pi_{t-1} \quad - \quad \rho(1-\theta)\tau_{t-1} \quad - \quad \frac{\rho\theta}{\alpha}\nu_{t-1} \quad + \quad \varepsilon_t \quad - \quad \tau_t \qquad \textbf{(4.15)}$$

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52 In the terminology of Bullard and Schaling (2000), variables such as last period's acceleration in inflation ($\Delta\pi_{t-1}$), last period's aggregate demand (y_{t-1}) and the update of past potential output (y_{t-2}^*) become *leading indicators* of supply side developments and will therefore also influence the current monetary policy stance.

First of all, both last period's persistent shock to potential output (τ_{t-1}) and last period's transitory cost-push shock (ν_{t-1}) will influence the output gap *indirectly* through their effect on the rate of inflation in the previous period (π_{t-1}). As in Section 4.2, the impact of this indirect effect will be influenced by the central bank's relative weight on output stabilisation (λ). Secondly, there will also be a *direct* effect of these shocks on the output gap because for a *given* value of the inflation forecast ($E_{t-1}\pi_t$) the central bank will try to move aggregate demand in step with its forecast of aggregate supply ($E_{t-1}y_t^*$). In this respect, a positive realisation of last period's shock to potential output will only be partly recognised by the central bank (the extent to which this is the case is measured by the signal-to-noise ratio θ). This means that aggregate demand will not fully reflect the size of this shock because of which it will induce a decrease in today's output gap. Similarly, a positive realisation of last period's *transitory* cost-push shock will be partly interpreted as a decrease in potential output. This will cause the central bank to contract aggregate demand and therefore also the output gap since this shock does not *actually* influence potential output itself.

It should also be noted that the central bank's relative weight on output stabilisation does *not* influence the *direct* effect of these shocks in any way. The reason for this is that the direct effect affects the output gap through the central bank's estimate of potential output. The central bank's relative weight on output stabilisation only affects the *trade-off* between inflation and *output gap* variability. The fact that the central bank tries to minimise the latter by allowing demand to grow in line with its estimate of potential ouput does not affect this trade-off in any way.

Plugging equation **(4.15)** back into the Phillipscurve equation **(4.1)** we find that inflation in period t will be equal to:

$$\pi_t = \frac{\lambda}{\lambda+\alpha^2}\pi_{t-1} - \alpha\rho(1-\theta)\tau_{t-1} - \rho\theta\nu_{t-1} - \alpha\tau_t + \alpha\varepsilon_t + \nu_t \qquad \textbf{(4.16)}$$

Since the output gap is one of the variables driving inflation, the afore-mentioned effects of last period's shock to potential output (τ_{t-1}) and the cost-push shock in the previous period (ν_{t-1}) will equally apply to the current rate of inflation.

From the aggregate demand equation **(4.2)** it can be seen that $i_t = - E_{t-1}y_t + E_{t-1}\pi_{t-1}$. Using equations **(4.15)** and **(4.16)**, the optimal nominal interest rate in period t will be equal to:

$$i_t = \frac{\alpha(\lambda+\alpha^2)+\lambda^2}{(\lambda+\alpha^2)^2}\pi_{t-1} - \rho^2 y^*_{t-2} + \frac{\rho\theta}{\alpha}\left[-\alpha\tau_{t-1}+v_{t-1}\right] \qquad \textbf{(4.17)}$$

When comparing this equation with the optimal interest rate obtained when there is no uncertainty about potential output it can be seen that the interest rate will also react expected supply side developments. In particular, the interest rate will be lower than the one obtained in equation **(4.7)** if the central bank expects potential output to be above its long term trend.

Next, we will consider the question whether or not the central bank should be more conservative if it faces uncertainty about potential output. From equations **(4.15)** and **(4.16)** we can see that $E(\pi) = E(y-y^*) = 0$ so minimising the social loss function with respect to λ again boils down to examining the trade-off between inflation and output gap variability. Before we start analysing this formally it is instructive to consider what might cause this trade-off to be different in the face of uncertainty about potential output. Comparing the expressions for inflation and the output gap without and with uncertainty about potential output (i.e. comparing equation **(4.5)** and **(4.6)** to equations **(4.15)** and **(4.16)**) we can see that the trade-off between inflation and output gap variability may be different because of the *interaction* between the direct and indirect effects of last period's shock to potential output and last period's cost-push shock. In other words, what makes the trade-off different from the one examined in Section 4.2 is that it will be influenced by the unconditional covariance between τ_{t-1} and v_{t-1} on the one hand, and π_{t-1}, on the other. After all, the indirect effect itself is incorporated into π_{t-1} and in this respect there is no difference with the case where there is no uncertainty about potential output. Next, as argued before, the direct effect taken on its own will not influence the trade-off either since the central bank's relative weight on output stabilisation has no impact on this.

Using equations **(4.15)** and **(4.16)** the variances of inflation and output will be equal to (see Appendix C):

$$Var(\pi) = \frac{(\lambda+\alpha^2)^2}{(2\lambda+\alpha^2)}\left[\left(1+\rho^2(1-\theta)^2\right)\sigma_\tau^2 + \frac{(1+\rho^2\theta^2)}{\alpha^2}\sigma_v^2 + \sigma_\varepsilon^2\right]$$

$$Var(y-y^*) = \left(1+\frac{\alpha^2}{(2\lambda+\alpha^2)}\right)\left[\left(1+\rho^2(1-\theta)^2\right)\sigma_\tau^2 + \frac{(1+\rho^2\theta^2)}{\alpha^2}\sigma_v^2 + \sigma_\varepsilon^2\right] - \frac{\sigma_v^2}{\alpha^2} \qquad \textbf{(4.18)}$$

Obviously, due to the variability in potential output and the central bank's inability to make a perfect ex ante distinction between *persistent* movements in potential output and *transitory* cost-push shocks, both the variance of inflation and the variance of the output gap will be larger than in Section 2. This can be seen by setting the parameters ρ and σ_τ^2 equal to zero in equation **(4.18)** in which case we obtain equation **(4.9).**

Plugging these expressions into the social loss function **(4.8)** and minimising the resulting expression with respect to λ we obtain *exactly* the same optimal value for the central bank's relative weight on output stabilisation as in the case where there is no uncertainty about the level of potential output. Hence, under an accelerationist Phillipscurve and optimal signal extraction by the central bank concerning movements in potential output, uncertainty about the latter will not warrant any *additional* degree of conservatism compared to the case where potential output is fixed and known. The intuition for this is that the long run effects of the interaction between the direct and indirect effects of the innovation in potential output and the cost push shock on the variability of inflation and the output gap will exactly cancel out. For instance, in Appendix C we show that the variance of inflation will be influenced by the unconditional covariances between τ_{t-1} and v_{t-1}, on the one hand, and π_{t-1} on the other. These covariances will exert opposite effects. In particular, $Cov(\tau_{t-1}, \pi_{t-1}) = -\alpha^2\sigma_\tau^2$ will have a positive effect on $Var(\pi)$ since π_{t-1} affects π_t positively while τ_{t-1} has a negative effect on π_{t-1} (see equation **(4.16)**). Similarly, $Cov(v_{t-1}, \pi_{t-1})=\sigma_v^2$ will have a negative effect on $Var(\pi)$. The effect of these covariances on the variability of inflation is influenced by the fact that the central bank optimally filters out information about supply side developments. For instance, an increase in σ_τ^2 will increase $Cov(\tau_{t-1}, \pi_{t-1})$. Due to the fact that the central bank is engaged in optimal signal extraction, it will *similtaneously* induce a decrease in the effect of this covariance on Var(π). Hence, the fact that these covariances are weighed by the signal-to-noise ratio θ ensures that their effect on Var(π) cancels out exactly.

The economic intuition behind this result is that optimal signal extraction in conjunction with the fact that the process driving potential output is stationary will ensure that the central bank will not make any *systematic* mistakes in its estimates of potential output. Since we assumed that the optimal relative weight on output stabilisation is chosen from an *ex ante* perspective (i.e. it will be determined by the *unconditional* variances of inflation and the output gap), uncertainty about potential output put will therefore have no effect on its optimal value.

4.4: Summary and Conclusion

In this chapter we have analysed a simple model of inflation targeting in which price setting is purely *backward looking* and in which output is therefore demand determined. The government instructs the central bank to minimise the variation of inflation around the assigned target and to minimise the variability of output around potential. Moreover, the government also explicitly assigns a relative weight on output stabilisation to the central bank. Since the central bank does not have an ambitious output target, there is no inflationary bias in this model in the sense that inflation will not deviate from the target systematically. At first sight it may seem that the government should therefore appoint a central banker who shares society's preferences concerning output stabilisation. However, as in Clarida, Gali and Gertler (1999), who analyse the case of purely *forward-looking* price setting, we find that it will be optimal from society's point of view to appoint a central banker who is conservative in the Rogoff (1985) sense. The reason is that the accelerationist Phillipscurve introduces a link between periods which is something a central banker acting under discretion will not take into account when setting the interest rate. Moreover, we relate the optimal degree of conservatism to some underlying structural economic parameters.

Next, we assume that the central banker is uncertain about the level of potential output when setting monetary policy. As described by for instance Rudebusch (2000), this corresponds to the situation faced by real world central bankers who, among other things, base their instrument settings on current estimates of supply side developments which are frequently revised at a later date. We then explore the question whether this kind of uncertainty will alter the optimal degree of conservatism compared to the case where potential output is fixed and known. In this respect we find that optimal signal extraction by the central bank concerning supply side developments will not alter the balance between the unconditional variances of inflation and the output gap. Therefore, the optimal relative weight on output stabilisation will be exactly the same as the one obtained when there is no uncertainty about potential output.

Nevertheless, we would like to emphasise that one should be very careful in drawing conclusions from this result for real world monetary policy. In particular, this result will be less useful when there is a suspicion that there may be a structural break in the time series of potential output.[53] Such a structural break occurred in the early 1970's in which the Western world experienced a significant decrease in productivity growth. It has been argued that the failure of policymakers to recognise this and the concomitant overestimation of potential output was partly responsible for the high rates of inflation experienced in this period (see e.g. Orphanides (2000)). Similarly, supply side developments may have undergone a significant change since the mid 1990's, even though no unequivocal conclusion can be drawn about this at this point in time. Following Orphanides (2000), we feel that it may very well be true that the concern for output stabilisation should decrease in the face of such structural breaks.

Next, our model assumes that the relative weight on output stabilisation is determined ex ante (i.e. in the institutional design phase) and that it will remain fixed and known to the public forever after that. As argued by Cukierman (2000) this may not be entirely realistic since many central bank's remain opaque about this parameter of their loss function and, moreover, since this parameter may change over time (see Cukierman and Meltzer (1986)). The latter may be the result of a continuously shifting balance of power within the central bank council

[53] Our model clearly does not take account of this possibility as can be seen from equation **(4.14)**.

but following our discussion earlier it may also be a response to changing circumstances in the economy. In our view, it would be certainly be interesting to investigate the consequences of opaqueness about the relative weight on output stabilisation and/or the possibility for this parameter to vary over time for the optimal degree of conservatism.

Appendix A: Proof of Proposition 4.1

First we will prove that $\lambda^* < \xi$ which using equation **(4.10)** boils down to the following condition:

$$-\alpha^2 + \alpha\sqrt{\alpha^2 + 4\xi} \; < \; 2\xi \; \Leftrightarrow \; \alpha^2(\alpha^2 + 4\xi) \; < \; (2\xi + \alpha^2)^2 \; \Leftrightarrow \; 0 \; < \; 4\xi^2$$

Q.E.D.

Next, the partial derivatives of λ^* read as follows:

$$\frac{\partial \lambda^*}{\partial \alpha} = \frac{\left(\alpha - \sqrt{\alpha^2 + 4\xi}\right)^2}{2\sqrt{\alpha^2 + 4\xi}} > 0 \qquad ; \qquad \frac{\partial \lambda^*}{\partial \alpha} = \frac{\alpha}{\sqrt{\alpha^2 + 4\xi}} > 0$$

Appendix B: Derivation of Equation (4.12)

From equation **(4.11)** we have:

$$E_{t-1} y_t^* \; = \; \rho^2 y_{t-2}^* \; + \; E_{t-1}\tau_{t-1} \tag{B.1}$$

While the central bank does not have a direct observation on τ_{t-1}, it does observe last period's inflation residual (Ω_{t-1}) which is equal to:

$$\Omega_{t-1} \; = \; -\alpha\tau_{t-1} + \nu_{t-1} \; = \; \pi_{t-1} - \pi_{t-2} - \alpha y_{t-1} + \alpha\rho y_{t-2}^* \tag{B.2}$$

Using equation **(B.2)** the central bank's optimal estimate for τ_{t-1} will be equal to:

$$E_{t-1}\tau_{t-1} \quad = \quad \frac{Cov_{t-1}(\Omega_{t-1}, \tau_{t-1})}{Var_{t-1}(\Omega_{t-1})}\Omega_{t-1} \quad = \quad \frac{-\alpha\sigma_\tau^2}{\alpha^2\sigma_\tau^2 + \sigma_\nu^2}\Omega_{t-1} \tag{B.3}$$

Plugging this into equation **(B.1)** will yield equation **(4.12)** in the main text.

Appendix C: Derivation of Equation (4.18)

From equation **(4.16)** we can see that inflation follows a stationary AR(1) process and that the covariance between all the other shocks affecting inflation will be equal to zero. Taking the square of equation **(4.16)** and applying the unconditional expectations operator to both sides of the equation we obtain:

$$\begin{aligned} E(\pi_t)^2 = {} & \frac{\lambda^2}{(\lambda+\alpha^2)}E(\pi_{t-1})^2 + \alpha^2(1+\rho^2(1-\theta)^2)\sigma_\tau^2 + (1+\rho^2\theta^2)\sigma_\nu^2 + \alpha^2\sigma_\varepsilon^2 \\ & -\frac{2\alpha\lambda\rho(1-\theta)}{(\lambda+\alpha^2)}E(\pi_{t-1}\tau_{t-1}) - \frac{2\lambda\rho\theta}{(\lambda+\alpha^2)}E(\pi_{t-1}\nu_{t-1}) \end{aligned} \tag{C.1}$$

Here we have used the fact that $E(\tau_{t-1})^2 = E(\tau_t)^2 = \sigma_\tau^2$, $E(\nu_{t-1})^2 = E(\nu_t)^2 = \sigma_\nu^2$ and $E(\varepsilon_t)^2 = \sigma_\varepsilon^2$. Next, lagging equation **(4.16)** one period it can be seen that $E(\pi_{t-1}\tau_{t-1}) = -\alpha\sigma_\tau^2$ and that $E(\pi_{t-1}\nu_{t-1}) = \sigma_\nu^2$. Using the expression obtained for θ in equation **(B.3)**, the last two terms on the RHS of equation **(C.1)** can be rewritten as follows:

$$\frac{2\lambda\rho}{(\lambda+\alpha^2)}[\alpha^2(1-\theta)\sigma_\tau^2 - \theta\sigma_\nu^2] \quad = \quad 0 \tag{C.2}$$

Using this in equation **(C.1)** and realising that $E(\pi_t)^2 = E(\pi_{t-1})^2 = Var(\pi)$ we obtain the first expression in equation **(4.18)**. Moreover, Var(y) can be obtained by applying a similar exercise.

Chapter 5: The Optimal Degree of Output Stabilisation under An Expectations-Augmented Non-Linear Phillipscurve

5.1: Introduction

Recently, the literature has seen a revival of the idea that the economy's aggregate supply relationship may be non-linear. In a nutshell, because of capacity constraints booms may be more inflationary than recessions are disinflationary. If this is the case, it may have important implications for monetary policy. Recent contributions in this respect include Clark, Laxton and Rose (1995), Bean (2000) and Schaling (1998). Clark et al. analyse the implications of several exogenous back- and forward-looking policy reactions functions. They show that in the case of an accelerationist convex Phillipscurve the mean level of output will be inversely related to output variability. This is because the increase in inflation as a result of a period of excess demand has to be compensated by a period of greater excess supply to disinflate the economy. Therefore, there will be an additional gain to output stabilisation if the central bank is faced with a convex accelerationist Phillipscurve. Next, they show there will be a strong case for pre-emptive strikes against inflation ('a stitch in time saves nine'). These results are confirmed by Bean (2000) who studies the optimal *endogenous* policy rule in a model where the output gap enters linearly in the central banker's loss function and where she observes a noisy signal of the demand shock. In particular, he shows that, compared the linear case, optimal policy will be more contractionary and that disinflations will be implemented more gradually. Schaling (1998) reaches similar conclusions by introducing a convex Phillipscurve in the Svensson (1997b) inflation forecast targeting framework for the case where the relative weight on output stabilisation is equal to zero.

In this chapter we will focus on the first result, i.e. we will study the question to what extent an endogenously derived (and hence, by definition, optimal) monetary policy rule will feature an additional return to output stabilisation compared to the linear case. As a benchmark, we first analyse the socially optimal relative weight on output stabilisation in a model with a linear expectations-augmented Phillipscurve. Contrary to the accelerationist Phillipscurve, the need to induce a negative output gap to squeeze an inflationary shock out of the economy will crucially depend on the central bank's credibility. We assume that the central bank cares about the variability of inflation around the assigned target and about the variability of output around potential. The latter assumption ensures that monetary policy will not be burdened by an inflationary bias of the Barro-Gordon type (see Barro and Gorden (1983)). Finally, we assume inflation is also influenced by a cost-push shock which is serially correlated. In the linear model we make a distinction between the case where the central bank has private information about the realisation of the cost-push shock and the case where there is no information asymmetry. If the central bank has private information it will be able to stabilise output and therefore a policy of strict inflation targeting will not be optimal. However, because the public can partially predict the cost-push shock we find that it will still be optimal to appoint a conservative central banker. Moreover, we will determine the determinants of the optimal degree of conservatism and contrast them with the results obtained in a situation where the central banker tries to push output above the long run natural rate systematically.

Under symmetric information the central bank will not be able to affect output at all. In other words, since the return to output stabilisation is zero in this case we find it will be optimal to appoint a central banker whose relative weight on output stabilisation is equal to zero as well. In Section 3, we analyse the implications of a convex Phillipscurve under symmetric information. As shown earlier by Schaling (1998), we find that a policy of strict inflation targeting (which is optimal in the linear case) will induce a deflationary bias. Essentially, this is because the convexity of the Phillipscurve causes the risks surrounding the optimal conditional inflation forecast to be asymmetric. However, whereas in the Schaling model the optimal inflation forecast will be constant over time we find that the presence of persistent cost-push shocks will cause this forecast to be state-dependent. As a result, in addition to demand uncertainty, uncertainty about inflationary shocks will induce a further downward bias in the long run average rate of inflation. This bias is also the reason why there is a social return to output stabilisation if the Phillipscurve is convex even if the central bank cannot affect output at all. This is because the long run average rate of inflation turns out to be strictly increasing in the central bank's relative weight on output stabilisation. As a result, the deflationary bias can be made less severe and welfare can be improved by the central bank's futile attempts to stabilise output.

5.2: The Optimal Degree of Conservatism with and without Private Information: the Linear Case

In this section we present an extenstion of one of the models studied by Svensson (1997a). The aggregate supply relationship is given by the expectations-augmented Phillips-curve:

$$\pi_t = \pi_t^e + \alpha y_t + \mu_t \qquad \textbf{(5.1)}$$

This equation can be seen as the inverse of the Lucas supply function. Consequently, whether or not monetary policy can affect output in the short-run depends crucially on the central bank's ability to generate a surprise inflation. In this model, where we abstract from uncertainty on the part of the public concerning central bank preferences, this boils down to the question whether or not the central bank has private information about the realisation of the cost-push shock (μ_t). The latter follows a serially correlated and stationary process where the innovation to the cost-push shock (v_t) follows an i.i.d. normal with mean zero and variance σ_v^2:

$$\mu_t = \rho\mu_{t-1} + v_t \qquad \textbf{(5.2)}$$

At this point is useful to make a distinction between the natural rate of output and potential output. As argued by Cukierman (2000), these two concepts are often used interchangeably but are not necessarily identical. The *natural rate of output* can be defined as the output level that will prevail in the absence of inflationary surprises. In this model the natural rate of output will therefore vary with the realisation of the cost-push shock. In particular, a positive cost-push shock will, in the absence of inflationary surprises, induce a decrease in the short-term natural rate which corresponds to a supply side induced decrease in the output gap.[54] By contrast, *potential* output corresponds to the *long-run* capacity of the economy to produce goods and services as determined by factors such as the capital stock, the labour force and the state of technology. In other words, this concept pertains to the level of output that will prevail in the absence of inflationary surprises *and* transitory cost-push shocks.[55] In this model, potential output is time-invariant and normalised to zero. Furthermore, since the Phillipscurve is linear and since we assume that cost-push shocks follow a stationary process, there will be no systematic deviation of the natural rate of output from potential.

Next, π_t^e denotes the rational expectation of inflation in period t conditional on all information available to the public at the end of period t-1. Throughout we assume that the public does not know the realisation of the supply shock but, due to the fact that this shock is serially correlated, can predict its realisation partially. As far as the central bank's information set is concerned we will distinguish between two cases. In the first case the central bank has an information advantage in the sense that it knows the realisation of the innovation to the supply shock (ν_t) when setting monetary policy. In the second case, which will serve as a benchmark for the analysis in Section 3, we analyse the situation where there is no information asymmetry.

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[54] Note that equation **(5.1)** can be rewritten as follows: $\pi_t = \pi_t^e + \alpha((y_t - y_t^n) + (y_t^n - y^*)) + \mu_t$ where y_t^n denotes the short-run natural rate and y^* denotes potential output (the latter is implicitly normalised to zero throughout this chapter). If there is no surprise inflation (i.e. if $\pi_t=\pi_t^e$) actual output (y_t) will *by definition* be equal to the short-term natural rate which implies: $y_t^n=-\mu_t/\alpha$. The output gap is defined as the difference between actual output (y_t) and potential output (y_t^*). Hence, actual output will be determined by a combination of demand and supply (i.e. cost-push) factors. However, since this is essentially a flex-price model (at least as far as prices on the goods market are concerned), demand factors will only be able to influence output (i.e. to generate a deviation of y_t from y_t^n) to the extent that they will induce a deviation of the actual rate of inflation (π_t) from the rate of inflation expected by the public (π_t^e).

[55] According to Cukierman (2000) the potential level of output is the level of output that will prevail in the absence of real business cycle effects. In our model this real business cycle effect corresponds to the cost-push shock since this shock will generate supply side induced variations in actual output.

Aggregate demand (y_t) is given by the following equation:[56]

$$y_t \;=\; -(i_t - \pi_t^e) \;+\; \varepsilon_t \tag{5.3}$$

First of all, demand is decreasing in the real interest rate (i_t-π_t^e). For analytical convenience we assume that aggregate demand is influenced by the same expected rate of inflation that features in the Phillipscurve. Next, demand is also influenced by a demand shock (ε_t) which follows an i.i.d. normal distribution with mean zero and variance σ_ε^2. To approximate the effect of lags in monetary policy we assume this shock is unobservable to both the central bank and the public when they respectively set the nominal interest rate and the expected rate of inflation.

The central bank's objective is to stabilise inflation around the target assigned by the government and to stabilise output around potential where the latter is normalised to zero:[57]

$$L_t \;=\; 2(1-\delta)E_t\left[\sum_{j=t}^{\infty}\delta^{j-t}\left[\frac{1}{2}\pi_j^2 \;+\; \frac{\lambda}{2}y_j^{\,2}\right]\right] \tag{5.4}$$

5.2.1: Central Bank has Private Information

Since we assume that the central bank cannot commit and since private agents cannot distill any additional information from past policy actions, the central bank's problem boils down to

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[56] Since the demand shock (ε_t) is by assumption not incorporated into inflationary expectations, actual output will essentially always be demand determined. Moreover, potential output is fixed and normalised to zero. Hence throughout this chapter the variable y_t will denote three concepts: aggregate demand, actual output and the output gap.

[57] This assumption is made in most of the literature. Alternatively, we could assume the central bank tries to stabilise output around the short-term or long-term natural rate. The latter will in the linear model be equal to potential output. However, if the Phillipscurve is convex, the long-run natural rate will be below potential output but will still be constant. For an analysis of a state-contingent output target which is equal to the short-term natural rate see Svensson (1997a).

minimising the period loss function subject to the Phillipscurve constraint. This yields the familiar first order condition:

$$E_{t-1}y_t \quad = \quad -\frac{\alpha}{\lambda}E_{t-1}\pi_t \tag{5.5}$$

To obtain solutions for inflation and output in the case where the central banker has an information advantage we take expectations conditional on the central bank's information set in period t-1 across the Phillips curve equation **(5.1)** and subsequently substitute the expression obtained for $E_{t-1}\pi_t$ into equation **(5.5)** which yields the following expression for the central bank's optimal expected value of the output gap:

$$E_{t-1}y_t \quad = \quad -\frac{\alpha}{\lambda+\alpha^2}\left[\pi_t^e + \mu_t\right] \tag{5.6}$$

Plugging this expression into the Phillipscurve equation **(5.1)**, taking rational expectations across the resulting expression based on the *public's information set* in period t-1 (i.e. *excluding* ν_t) we obtain the equilibrium expected rate of inflation:[58]

$$\pi_t^e \quad = \quad \frac{\lambda\rho}{\alpha^2}\mu_{t-1} \tag{5.7}$$

Since the supply shock is to some extent predictable to the public, part of the central bank's reaction to it is anticipated and incorporated into inflationary expectations. This in turn will inhibit the central bank's attempts to stabilise output. However, since the central bank follows a discretionary policy it will not take the effect of its stabilisation efforts on expected inflation into account. In other words, the cental bank is faced with a time-inconsistency problem. Given the fact that its efforts to stabilise the effect of the *anticipated* part of the cost-push shock ($\rho\mu_{t-1}$) on output will be futile, it would be better if the central bank could commit to not reacting to this part of the cost-push shock. However, if the public were to believe this, the

[58] In the terminology of Svensson (1997a) this equation represents a state-contingent inflation bias.

central bank would have an incentive to cheat. Using equation **(5.6)** and **(5.7)** in the aggregate demand equation **(5.3)** yields the optimal interest rate in period t:

$$i_t \quad = \quad \left[\frac{\lambda+\alpha}{\alpha^2}\right]\rho\mu_{t-1} \quad + \quad \frac{\alpha}{\varphi(\lambda+\alpha^2)}\nu_t \tag{5.8}$$

The effect of the central bank's relative weight on output stabilisation on the nominal interest rate will be ambiguous. On the one hand, due its effect on inflationary expectations, a larger relative concern for output stabilisation will induce a stronger reaction to the realisation of the supply shock in period t-1 (μ_{t-1}). On the other hand, the central bank's reaction to the innovation of the supply shock (ν_t) will be less strong if it attaches more weight to output stabilisation.

Next, substituting the expected rate of inflation back into equation **(5.1)** and realising that $y_t = E_{t-1}y_t + \varepsilon_t$ we can compute the equilibrium output gap in period t:

$$y_t \quad = \quad -\frac{\rho}{\alpha}\mu_{t-1} \quad - \quad \frac{\alpha}{\lambda+\alpha^2}\nu_t \quad + \quad \varepsilon_t \tag{5.9}$$

The first term on the RHS of this equation is simply a reflection of the fact that a positive realisation of the cost-push shock (or equivalently a negative supply shock) which is fully anticipated by the public will decrease the natural rate in period t. The second term on the RHS shows that the effect of the innovation to the supply shock on output can be diminished by the central bank's stabilisation efforts by virtue of the fact ν_t will not be incorporated into inflationary expectations. Finally, the third term reflects the central bank's imperfect control over the output gap.

Next, by inserting equations **(5.7)** and **(5.9)** into the Phillips-curve we obtain the following equilibrium solution for inflation in period t:

$$\pi_t = \frac{\lambda\rho}{\alpha^2}\mu_{t-1} + \frac{\lambda}{\lambda+\alpha^2}v_t + \alpha\varepsilon_t \tag{5.11}$$

The first expression on the RHS is equal to the expected rate of inflation. Obviously, the central bank would do better not to react to the part of the supply shock which is also known to the public but cannot commit to doing so. The second part on the RHS represents the effect of the innovation to the supply shock where we can see that the degree to which this shock will feed through into inflation will be an increasing function of the central bank's relative weight on output stabilisation (λ). Finally, the last expression on the RHS denotes the effect of the unexpected demand shock on inflation.

The fact that the central bank tries to stabilise the effect of the supply shock on output but is partly prevented from doing so due to the fact that the supply shock is partially predictable begs the question as to what extent the central bank should seek to stabilise output in the first place. To evaluate this question we need the unconditional variances of inflation and output which are equal to:

$$\begin{aligned} Var(\pi) &= \left[\frac{\lambda^2\rho^2}{\alpha^4(1-\rho^2)} + \frac{\lambda^2}{(\lambda+\alpha^2)^2}\right]\sigma_v^2 + \alpha^2\sigma_\varepsilon^2 \\ Var(y) &= \left[\frac{\rho^2}{\alpha^2(1-\rho^2)} + \frac{\alpha^2}{(\lambda+\alpha^2)^2}\right]\sigma_v^2 + \sigma_\varepsilon^2 \end{aligned} \tag{5.12}$$

The optimal value of λ can now be obtained by inserting the expressions in equation **(5.12)** into the unconditional expected value of the social loss function:

$$E(L) = Var(\pi) + (E(\pi))^2 + \xi\left[Var(y) + (E(y))^2\right] \tag{5.13}$$

Here the parameter ξ denotes the socially optimal relative weight on output stabilisation. Minimising equation **(5.13)** with respect to λ we can derive the following proposition:

Proposition 5.1:

If the central bank is faced with a linear expectations-augmented Phillipscurve, if it has private information concerning cost-push shocks and if the public can predict these shocks partially, it will be optimal to appoint a conservative central banker (i.e. a central banker for whom it holds that $0 < \lambda^* < \xi$) *even* if the central banker's output target is equal to the long run natural rate. The central banker's optimal relative weight on output stabilisation (λ^*) will increase if:

1. the slope of the Phillipscurve (α) increases
2. the persistence of supply shocks (ρ) decreases
3. society's relative weight on output stabilisation (ξ) increases

Proof: see Appendix A

Hence, the seminal result obtained by Rogoff (1985) that society gains from appointing a central banker who is more inflation averse than society does not necessarily require a temptation to systematically push output above potential provided supply shocks are partially anticipated by the public. The reason for this result resides in the fact that, even though the choice of λ does not affect the unconditional expectations of inflation and output[59], it *does* affect their variances. In particular, an increase in the relative weight on output stabilisation will decrease the variance of output at the expense of an increase in the variance of inflation. Intuitively, in the presence of such a trade-off not paying *any* attention to output stabilisation cannot be optimal.

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[59] The latter is of course a direct result of the natural rate hypothesis implying no systematic effect of monetary policy (including the central bank's preference parameters) on output. Furthermore, at the risk of repetition, it should be noted that the long run natural rate of output E(y) is equal to potential output.

Next, the fact that the optimal degree of output stabilisation is lower than society's preference for output stabilisation stems precisely from the fact that the public can partially predict supply shocks. This creates a problem which is similar in spirit to the well-known inflationary bias problem even though such a bias is not present in this model.[60] The central bank's efforts to limit the impact of the *predictable* part of the supply shock on output will in equilibrium only affect the (expected) rate of inflation. This means that, as far as its reaction to the predictable part of the supply shock is concerned, the central bank will end up injecting more inflation variability into the economy without reducing output variability.[61] Hence, society would be better off if the central bank were able to commit to not reacting to the predictable part of the supply shock.[62] In exactly the same spirit as in the Rogoff (1985) model, in the absence of a commitment mechanism society gains from appointing a conservative central banker.

Next, we will examine the determinants of the optimal degree of conservatism and compare them with the results obtained by Eijffinger and Hoeberichts (1998). These authors analyse the optimal degree of conservatism in a framework where the central bank's output target exceeds the long run natural rate and in which there is no persistence in supply shocks. In that case there is a trade-off between *credibility* and *flexibility*. In other words, by appointing a more conservative central banker, society will gain since this will lower the *first moment* of the unconditional distribution of inflation towards society's bliss point. However, this will come at the cost of an increase in the *second moment* of the unconditional distribution of output which will be suboptimal since output will no longer be stabilised in accordance with social preferences. We emphasise that in our model society is concerned with a trade-off between the *second* moments of *both* distributions. The comparison is summarised in Table 5.1.

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[60] Throughout this chapter we will define an inflationary/deflationary bias as a positive/negative deviation of the *unconditional* expectation of inflation from the target.

[61] The variability in inflation caused by the central bank's reaction to $\rho\mu_{t-1}$ corresponds to the first term on the RHS in equation **(5.12)**.

[62] Note that such a commitment would not be credible for exactly the same reasons a commitment to a zero-inflation rule is not credible in models dealing with the inflationary bias problem in monetary policy (see e.g. Barro and Gordon (1983), Rogoff (1985) and Lohmann (1992)).

Table 5.1: Effect of Model Parameters on the Optimal Degree of Conservatism[63]

	Effect on Optimal Degree of Conservatism with Output Target *Exceeding* Long Run Natural Rate (see Eijffinger and Hoeberichts (1998)	Effect on Optimal Degree of Conservatism with Output Target *equal* to Long Run Natural Rate (this model)
α	- if ξ relatively high + if ξ relatively low	-
ρ	0	+
ξ	+	-
σ_v^2	+	0

First of all, the parameter α can be seen as the inverse of the slope of the Lucas supply function.[64] In the model with an ambitious output target, a decrease in α would increase the effect of any given surprise inflation on output supply. Eijffinger and Hoeberichts (1998) show that this will increase the temptation for the central banker to generate surprise inflation if society already attaches a relatively high relative weight to output stabilisation. Since this will aggravate society's credibility problem (i.e. it will increase the inflationary bias) the central bank needs to be more conservative. However, in the model analysed in this paper there is no *systematic* temptation to spring surprise inflation on the public. Consequently, there is no long run inflationary bias but instead there will be excessive *inflation variability* if the central banker inherits society's preferences. The very fact that central banker cares about output stabilisation ($\lambda > 0$) will cause the rate of inflation to be influenced by supply shocks. However, given this dependence, an increase in the slope of the Phillipscurve (α) will *diminish*

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[63] Eijffinger and Hoeberichts (1998) abstract from persistence in supply shocks. However, based on the results obtained by Svensson (1997a), it can be argued that in the model with an ambitious output target the optimal degree of conservatism will be increasing in the persistence of supply shocks as well.

[64] Note that the Lucas supply function corresponding to equation **(5.1)** would be: $y_t = \frac{1}{\alpha}\left(\pi_t - \pi_t^e - \mu_t\right)$

the variability of inflation which is caused by the variance of supply shocks (σ_v^2) as can be seen from equation (20). As a result there is more room for the central banker to stabilise the effect of supply shocks on output.

Next, an increase in the persistence of supply shocks (ρ) will increase the volatility of inflationary expectations and therefore also the variability of inflation caused by the central bank's futile attempts to stabilise the effect of the predictable part of supply shocks on output. These attempts were earlier identified to be the cause of the fact that society gains by appointing a conservative central banker. Hence, if the effect of the central bank's reaction to predictable supply shocks on the variance of inflation increases, it will be optimal from society's point of view to pay less attention to output stabilisation.

Third, if society shows an increased concern for output stabilisation (ξ) in the model with an ambitious output target, a policymaker sharing society's preferences will produce a higher inflationary bias. Hence, society's credibility problem will become more severe and this needs to be counteracted by an increase in the degree of conservatism. In the present model, the effect of an increase in ξ is precisely the reverse. The reason is that there is no temptation whatsoever to push output above the long run natural rate systematically. The only relevant thing at stake here is the trade-off between inflation and output *variability*. Hence, if society displays an increased dislike of output variability relative to inflation variability, this will spill over into a decrease in the optimal degree of conservatism.

Finally, an increase in the variance of supply shocks (σ_v^2) will reduce the optimal degree of conservatism in the model with an ambitious output target because it aggravates society's flexiblity problem relative to its credibility problem. In our model the variance of supply shocks drops out of the first-order condition, and has consequently no effect on λ^*. Apparently, given that the degree of conservatism is optimal to start with, an increase in σ_v^2 will not alter the balance between the marginal benefit of an increase in the degree of conservatism (stemming from lower inflation variability) and the marginal cost of such an increase (stemming from higher output variability).

5.2.2: Central Bank has no Private Information

In the next section where we analyse the case of a non-linear Phillipscurve we will drop the assumption that the central bank has private information in order to be able to obtain analytical results. As a benchmark it is therefore instructive to analyse the outcome in the linear model where the central bank has no private information. In particular we will assume that the realisation of the *innovation* to the supply shock is not known to the central bank when setting monetary policy. Since the computations are in the same spirit as in the private information case we will restrict ourselves to presenting the outcomes:

$$\pi_t^e = \frac{\lambda\rho}{\alpha^2}\mu_{t-1}$$

$$\pi_t = \frac{\lambda\rho}{\alpha^2}\mu_{t-1} + \nu_t + \alpha\varepsilon_t$$

$$y_t = -\frac{\rho}{\alpha}\mu_{t-1} + \varepsilon_t$$

$$i_t = \left[\frac{\lambda+\alpha}{\alpha^2}\right]\rho\mu_{t-1}$$

$$Var(\pi) = \left[\frac{\lambda^2\rho^2}{\alpha^4(1-\rho^2)}+1\right]\sigma_\nu^2 + \alpha^2\sigma_\varepsilon^2$$

$$Var(y) = \frac{\rho^2}{\alpha^2(1-\rho^2)}\sigma_\nu^2 + \sigma_\varepsilon^2 \qquad \textbf{(5.14)}$$

Examining the expressions for the variance of inflation and output, we obtain the following proposition:

Proposition 5.2:

If the central bank is faced with a linear-expectations augmented Phillipscurve, if there is no information asymmetry and if both the central bank and the public can partially predict supply shocks, it will be optimal to appoint an ultra-conservative central banker who only cares about inflation stabilisation (i.e. $\lambda^* = 0$) even if the central bank has an output target which is equal to potential output.

The proof of this proposition is relatively straightforward since it can easily be seen that the variance of inflation is strictly increasing in the central bank's relative weight on output stabilisation while the variance of inflation is not affected by λ. Obviously, minimising the social loss function **(5.13)** then boils down to minimising the variance of inflation with respect to λ.

This result should not be surprising since it is exactly the same as the one obtained in the Rogoff (1985) model if it is assumed that the central bank has no private information about the supply shock. The reason is relatively simple, if the central bank cannot affect output while its attempts to do so will affect (expected) inflation (and hence inflation variability), it will be optimal to abstain from output stabilisation entirely. In that case, the only thing the central bank will do is to offset the impact of the *predictable* part of the supply shock on inflation completely by setting the interest rate such that expected demand equals expected supply ($-\rho\mu_{t-1}$) at the point where the inflation forecast is equal to the target. However, in contrast to the Rogoff model, it should be noted that the optimality of an ultra-conservative central banker in this case critically depends on the fact that the cost-push shock is partially predictable to the public. If this is not the case (i.e. if $\rho = 0$), the central bank's relative weight on output stabilisation will become irrelevant in the sense that is will not affect the variance of inflation and output

5.3: Optimal Monetary Policy with a Non-Linear Expectations-Augmentend Phillipscurve

In this section we will analyse the optimal degree of output stabilisation when the central bank is faced with a non-linear Phillipscurve. In particular, we assume that for given values of expected inflation and the supply shock, the inflation-output trade-off ($f(y_t)$) becomes more unfavourable as the output gap increases and, secondly, that an output gap which is equal to zero has no effect on inflation (i.e. we assume $f'(y) > 0$, $f''(y) > 0$ and $f(0) = 0$). To derive some analytical results we will use the following functional form which can be regarded as a local approximation to any arbitrary convex short-term inflation-output trade-off:[65]

$$\pi_t = \pi_t^e + \frac{e^{\chi\alpha y_t} - 1}{\chi} + \mu_t \tag{5.15}$$

Here the cost-push shock is given by equation **(5.2)**. The parameter χ indexes the *curvature* of the inflation-output trade-off. In particular, this function has the desirable properties that the curvature does not depend on the output gap and that we obtain the linear specification as χ approaches zero:

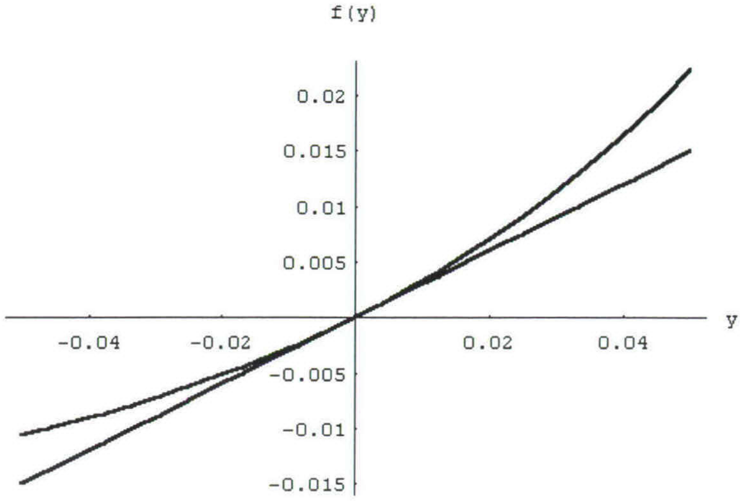
$$\lim_{\chi \downarrow 0} \frac{e^{\chi\alpha y_t} - 1}{\chi} = \alpha y_t \qquad \textit{and} \qquad \frac{f''(y_t)}{f'(y_t)} = \chi\alpha \tag{5.16}$$

To get some feel for the difference in the inflation-output trade-off induced by the non-linear specification we have drawn equations **(5.1)** and **(5.15)** in a diagram for α=0.3 and χ = 50:[66]

[65] Bean (2000) uses this formulation in empirical tests of the nonlinearity of the Phillipscurve.

[66] These are the values estimated by Bean (2000) for the United Kingdom.

Figure 5.1: Linear and Non-Linear Inflation-Output Trade-Off



Again, the expected value of the output gap conditional on the central bank's information set in period t-1 ($E_{t-1}y_t$) can be seen as an indirect control variable. The first-order condition for the central bank's problem then becomes:

$$E_{t-1}[\pi_t \gamma + \lambda y_t] = 0 \qquad where \qquad \gamma = \frac{\partial \pi_t}{\partial y_t} = \alpha e^{\chi \alpha y_t} \tag{5.17}$$

Here we define γ as the short-run inflation output trade-off which, in contrast to the certainty equivalent case, depends on the output gap and which, from the point of view of the central bank, is a stochastic variable.

Solving for the expectations operator (see Appendix B) we obtain the central bank's optimal expected value of the output gap as an implicit function of expected inflation:

$$E_{t-1}y_t = -\frac{E_{t-1}\gamma}{\lambda}E_{t-1}\pi_t - \frac{\chi\alpha^3}{\lambda}\sigma_\varepsilon^2 \qquad where \qquad E_{t-1}\gamma = \alpha e^{\chi\alpha E_{t-1}y_t}(1+\frac{(\chi\alpha)^2}{2}\sigma_\varepsilon^2) \quad \textbf{(5.18)}$$

In essence the central bank still pursues a 'leaning against the wind' policy by contracting demand if the conditional inflation forecast exceeds the target. However, as indicated by the second term on the RHS, uncertainty about the current inflation-output trade-off will cause the central bank to set the expected value of the output gap so as to 'err on the side of caution'. In other words, even if the inflation forecast is equal to the target (implying $E_{t-1}\pi_t = 0$) the central bank will contract output. The reason is that the risks surrounding the central forecast are not symmetric in the sense that a given absolute value of the realisation of the demand shock will be more inflationary if it is positive than deflationary when it is negative. Consequently, the optimal expected value of the output gap will be lower than in the certainty-equivalent case (see also Schaling (1998) and Bean (2000)). Obviously, the central bank will be more cautious if the degree of uncertainty (as measured by the variance of demand shocks) increases and if the curvature of the inflation-output trade-off increases. An increase in the relative weight on output stabilisation (λ) will diminish the degree of caution.

In order to obtain closed form solutions for $E_{t-1}y_t$ and $E_{t-1}\pi_t$ we use the Phillipscurve relationship **(5.15)** and take rational expectations on both sides of the equation. From this it follows that the conditional expected value of the output gap is restricted to be equal to (see Appendix B):[67]

$$E_{t-1}y_t \quad = \quad \frac{1}{\chi\alpha}\left[\ln(1-\chi\rho\mu_{t-1}) - \ln\left(1+\frac{(\chi\alpha)^2}{2}\sigma_\varepsilon^2\right)\right] \qquad \textbf{(5.19)}$$

This equation represents the short-term *expected* natural level of output and acts as an (expected) short run supply constraint for monetary policy. This is because the central bank

[67] Here it holds that $\lim_{\chi\downarrow 0} E_{t-1}y_t = -\frac{\rho}{\alpha}\mu_{t-1}$, i.e. as χ approaches zero the model will collapse into the linear symmetric information model (see equation **(5.14)**).

cannot *intentionally* spring a surprise inflation on the public.[68] Therefore, $E_{t-1}y_t$ will be determined by the predictable part of the supply shock and the variance of demand shocks, both of which will be incorporated into inflationary expectations. Since the Phillipscurve is convex, a given positive realisation of the supply shock will decrease output by a larger amount than a negative realisation of equal magnitude will increase output. Next, private agents know that *symmetric* risks surrounding the output gap (as measured by σ_ε^2) will be translated into *asymmetric* risks for expected inflation. This will induce an upward bias in expected inflation and will therefore decrease the expected value of the output gap compared to the certainty equivalent case.

Equation **(5.19)** also allows us to pin down the value of the expected inflation-output trade-off. In Appendix B this is shown to be equal to:

$$E_{t-1}\left[\frac{\partial \pi_t}{\partial y_t}\right] = E_{t-1}\gamma_t = \alpha\left(1-\chi\rho\mu_{t-1}\right) \qquad \textbf{(5.20)}$$

This equation shows that the fact that cost-push shocks are partially predictable will cause the *expected* slope of the Phillipscurve to be *state-contingent*. In particular, the conditional expected inflation-output trade-off will be more favourable (in the sense that inflation will be less sensitive to the output gap) in the presence of positive cost-push shocks. This is because the latter will induce a contraction in the expected output gap because of which the central bank expects the economy to be on a flatter part of the Phillipscurve.

Plugging equation **(5.20)** into equation **(5.18)** and equating the latter to the expected output gap dictated by the structure of the economy as given by equation **(5.19)** yields the following

[68] Of course, the actual level of output will be influenced by the demand shock (ε_t). However, the central bank does not know the realisation of this shock when setting monetary policy.

solution for the conditional expected rate of inflation:[69]

$$E_{t-1}\pi_t = \frac{\frac{\lambda}{\chi\alpha^2}\left(\ln\left(1+\frac{(\chi\alpha)^2}{2}\sigma_\varepsilon^2\right)-\ln\left(1-\chi\rho\mu_{t-1}\right)\right)-\chi\alpha^2\sigma_\varepsilon^2}{\left(1-\chi\rho\mu_{t-1}\right)} \qquad \textbf{(5.21)}$$

First of all, output gap uncertainty (captured by σ_ε^2) has two opposing effects on the equilibrium conditional expected rate of inflation. On the one hand, an increase in output gap uncertainty will induce the central bank to contract demand so as to 'err on the side of caution', i.e. ceteris paribus it will seek to achieve a lower value of $E_{t-1}y_t$ because of the asymmetric risks surrounding the conditional inflation forecast. This is captured by the last term on the RHS of equation **(5.21)**. On the other hand, an increase in output gap uncertainty, through its effect on inflationary expectations will induce a decrease in the conditional output gap dictated by the structure of the economy (i.e. it will make the short run supply side constraint more severe). As in the case of a linear Phillipscurve, the fact that the central bank also seeks to stabilise output induces it ceteris paribus to implement a more expansionary policy to offset this effect. However, since this is fully anticipated by the public the only result will be an increase the expected rate of inflation.

The second interesting feature about equation **(5.21)** is that even under strict inflation targeting (λ=0) optimal monetary policy will result in a *state-contingent* optimal conditional inflation forecast. This result stands in marked contrast to earlier analyses of optimal monetary policy in the case where the central bank only cares about stabilising inflation around the assigned target. In the case of a linear Phillipscurve, the optimal conditional inflation forecast will then simply be equal to the inflation target (see Svensson (1997b)). If the Phillipscurve is convex and if there is no persistence in the process driving cost-push shocks, the optimal conditional inflation forecast will also be constant over time but will be lower than the target because of the asymmetric risks surrounding it (see Schaling (1998)). In essence this is because the

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[69] Where it holds that $\lim_{\chi \downarrow 0} E_{t-1}\pi_t = \frac{\lambda\rho}{\alpha^2}\mu_{t-1}$ (see equation **(5.14)**).

expected slope of the Phillipscurve will be constant in that case. This can be seen by setting ρ equal to zero in equation **(5.20)**. In this model the expected inflation-output trade-off is a function of last period's cost-push shock. For instance, a positive realisation of this shock will reduce the expected slope of the Phillipscurve resulting in an increase in the optimal conditional inflation forecast. This is because the central banker now expects demand shocks to have a smaller effect on inflation. Because of this the need to hedge against the fact that a positive demand shock will be more inflationary than a negative demand shock of equal magnitude will be deflationary, will diminish.

Equations **(5.19)** and **(5.21)** also allow us to compute the optimal interest rate in period t:[70]

$$i_t = \left(\frac{1}{\chi\alpha}+\frac{\lambda}{\chi\alpha^2(1-\chi\rho\mu_{t-1})}\right)\left[\ln(1+\frac{(\chi\alpha)^2}{2}\sigma_\varepsilon^2)-\ln(1-\chi\rho\mu_{t-1})\right]-\frac{\chi\alpha^2\sigma_\varepsilon^2}{(1-\chi\rho\mu_{t-1})} \quad \textbf{(5.22)}$$

Intuitively, a non-linear Phillipscurve will result in a non-linear policy rule in which the reaction coefficients to the determinants of inflation are no longer constant but rather a function of these determinants themselves. Furthermore, the impact of output gap uncertainty (σ_ε^2) on the optimal interest rate is ambiguous since it has two opposing effects on the equilibrium expected value of the output gap.

Using equation **(5.21)** we can derive an expression for the unconditional expectation of the output gap by applying the law of iterated expectations and taking a 2nd order Taylor expansion around $E(\mu_{t-1})$ which yields:

$$E(y) = \frac{1}{\chi\alpha}\left(-\frac{(\chi\rho)^2\sigma_v^2}{2(1-\rho^2)} - \ln\left(1+\frac{(\chi\alpha)^2}{2}\sigma_\varepsilon^2\right)\right) \quad \textbf{(5.23)}$$

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[70] Where $\lim_{\chi\downarrow 0} i_t = \left[\frac{\lambda+\alpha}{\alpha^2}\right]\rho\mu_{t-1}$ (see equation **(5.14)**).

Since we assumed that potential output is fixed and normalised to zero, this expression can be interpreted as the *long-run* natural rate. As noted by Bean (2000) and Schaling (1998), the convexity of the Phillipscurve will cause the long run natural rate to be below potential output. In these models, this effect is entirely attributed to uncertainty about demand shocks (captured by σ_ε^2) which causes the contractionary bias in monetary policy described in equation **(5.18)** and corresponds to the last term on the RHS of equation **(5.23).** In addition, the current model shows that the presence of persistent cost-push shocks will cause a further decrease in the long run natural rate over and above the decrease caused by demand uncertainty. The reason is that realisations of the supply shock are symmetric around zero and will be translated into decreases of the output gap which on average will be larger than the increases.

Next, we can compute the unconditional expectation of inflation by applying the law of iterated expectations and taking a 2nd order Taylor expansion around $E(\mu_{t-1}) = 0$ in equation **(5.21):**

$$E(\pi) = \frac{\lambda}{\chi\alpha^2}\ln\left(1+\frac{(\chi\alpha)^2}{2}\sigma_\varepsilon^2\right) - \chi\alpha^2\sigma_\varepsilon^2 + \frac{\chi\rho^2\sigma_v^2\left(3\lambda - 2\chi^2\alpha^4\sigma_\varepsilon^2 + 2\lambda\ln\left(1+\frac{(\chi\alpha)^2}{2}\sigma_\varepsilon^2\right)\right)}{2\alpha^2(1-\rho^2)} \qquad \textbf{(5.24)}$$

One of the interesting implications of equation **(5.24)** is that the central bank's relative weight on output stabilisation (λ) affects the first moment of the long run (i.e. unconditional) distribution of inflation. It has long been recognised that this preference parameter could be one of the determinants of the well-established positive correlation between the unconditional mean and variance of inflation (see e.g. Cukierman (1992)). However, theoretical explanations for this usually rely critically on the assumption that the central bank has an ambitious output target. As shown by the analysis in Section 5.2, if this assumption is dropped the relative weight on output stabilisation will cease to influence the unconditional expected rate of inflation. Nevertheless, as indicated by equation **(5.24)**, the introduction of a non-linear Phillipscurve (or indeed, any deviation from the certainty equivalent framework) re-establishes the link between the unconditional expected rate of inflation and the central bank's relative weight on output stabilisation.[71] In particular, an increase in this preference parameter will also increase long run average rate of inflation (E(π)). The reason for this is twofold: First of all, abstracting from the presence of cost-push shocks, demand uncertainty (σ_ε^2) will cause the natural rate to systematically fall below potential output. Since the central bank tries to stabilise output around potential, its attempts to counteract this will result in a ceteris paribus higher long run average expected rate of inflation. Next, for a given degree of demand uncertainty, the central bank also tries to stabilise the effect of the predictable part of the cost-push shock on output. However, because of the convexity of the Phillipscurve, cost-push shocks which are distributed symmetrically around zero will on average result in a decrease in output below potential. Hence, the central bank's reaction to the cost-push shock will on average cause a further ceteris paribus increase in the average expected rate of inflation.

It is instructive to first consider the case where the central banks abstains from output stabilisation entirely (i.e. $\lambda = 0$) since this was shown to be optimal in the case where the central bank also does not have an information advantage over the public but is instead faced with a *linear* expectations-augmented Phillipscurve. In that case equation **(5.24)** simplifies into the following expression:

$$E(\pi)\big|_{\lambda=0} \quad = \quad - \quad \frac{\chi^3\alpha^2\rho^2\sigma_\nu^2\sigma_\varepsilon^2}{(1-\rho^2)} \quad - \quad \chi\alpha^2\sigma_\varepsilon^2 \qquad \textbf{(5.25)}$$

[71] Cukierman (2000) shows that an inflationary bias will also emerge under the realistic assumption that the central bank cares more about negative output gaps relative to positive ones even if the central bank does not seek to drive output above the natural rate systematically.

From this expression we can derive:

Proposition 5.3:

If the expectations-augmented Phillipscurve is non-linear and if there is no information asymmetry, a central bank which engages in strict inflation targeting (i.e. $\lambda = 0$) will induce a *deflationary bias* which will become more severe if:

1. the variance of demand shocks (σ_ε^2) increases
2. the variance of supply shocks (σ_v^2) increases
3. the curvature indexation parameter (χ) increases
4. the average slope of the Phillipscurve (α) increases
5. the persistence of supply shocks (ρ) increases

The proof of this proposition follows immediately from equation **(5.25).** First of all, the intuition for this proposition rests on the combination of uncertainty about demand (σ_ε^2) on the one hand, and the fact that booms are more inflationary than recessions are deflationary, on the other. The latter explains why parameters which increase the curvature of the Phillipscurve $(\chi\alpha)$ will cause a decrease in the long run average rate of inflation $(E(\pi))$. After all for a given level of demand uncertainty an increase in the curvature will enhance the skewness of risks surrounding the central inflation forecast. As a result, the central bank will step up the degree to which it hedges against these risks by aiming for a lower rate of inflation. Similarly, the central bank will also become more cautious if (for a given curvature of the Phillipscurve) the variance of demand shock increases.

The deflationary bias in equation **(5.25)** can be broken down in two parts. The first part, represented by the last term on the RHS of this equation, reflects the bias which would obtain if, due to the absence of persistence in cost-push shocks, the central bank were faced with a constant *expected* slope of the Phillipscurve. As argued before, in that case the optimal inflation target will be constant over time (see equation **(5.21)**). Next, the first part on the RHS of equation **(5.25)** represents that part of the deflationary bias which can be ascribed to the fact that the expected slope of the Phillipscurve changes over time. A state-contingent expected inflation-output trade-off will result in a more severe deflationary bias than would obtain in the absence of persistent cost-push shocks. A negative realisation of μ_{t-1} will cause a decrease in the optimal conditional inflation forecast, relative to the constant value it would have in the absence of persistent cost-push shocks (i.e. relative to $E(\pi)|_{\lambda=\rho=0} = \chi\alpha^2\sigma_\varepsilon^2$), which will be larger than the increase resulting from a positive realisation of μ_{t-1} of equal magnitude. In essence, this is because the central bank expects the slope of the Phillipscurve to be steeper than average if last period's cost push shock was negative. In that case the need to hedge against the afore-mentioned asymmetric risks surrounding the forecast will become more pressing. This also explains why an increase in the parameters ρ and σ_v^2 (alongside an increase in the curvature of the Phillipscurve) will cause the deflationary bias to become more severe since both will serve to increase the volatility of the expected slope of the Phillipscurve.[72]

In the previous section it was shown that it will be optimal to appoint a central banker who only cares about inflation stabilisation if both the central bank and the public can predict supply shocks partially. Given our results sofar it is interesting to see whether this claim still holds in the case of a non-linear Phillipscurve. To assess the effect of λ on welfare we start by noting that from equations **(5.20)** and **(5.23)** we can see that neither the mean nor the variance of output will be affected by this parameter. Hence, minimising the social loss function **(5.13)** with respect to λ boils down to minimising the Mean-Squared Error of inflation ($MSE(\pi)= E(\pi^2) = Var(\pi) + (E(\pi))^2$). Due to the non-linearity of the Phillipscurve, the value of $E(\pi^2)$ is very difficult to obtain analytically. However, the actual rate of inflation will only differ from its conditional forecast (given by equation **(5.21)**) because of shocks to which the central bank cannot react (i.e. because of v_t and ε_t). Therefore, we will use the MSE of the *conditional inflation forecast* ($E_{t-1}(\pi_t)$) as an approximation:[73]

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[72] From equation **(5.20)** it can be seen that $E_{t-1}\gamma_t$ is normally distributed with mean α and variance $(\chi\alpha\rho)^2\sigma_v^2$.

[73] This expression can be obtained by taking the square of equation **(5.21)** and computing the unconditional expectation of the resulting expression using a 2nd order Taylor expansion around $E(\mu_{t-1}) =0$.

$$E\left[(E_{t-1}\pi_t)^2\right] = -\frac{\chi\rho\sigma_v^2}{(1-\rho^2)}\left[\lambda\sigma_\varepsilon^2 - \frac{\lambda^2\ln(1+\frac{(\chi\alpha)^2}{2}\sigma_\varepsilon^2)^2}{(\chi\alpha^2)^2}\right] +$$

$$\left[1+\frac{\chi\rho\sigma_v^2}{(1-\rho^2)}\right]\left[\frac{\lambda^2\ln(1+\frac{(\chi\alpha)^2}{2}\sigma_\varepsilon^2)^2}{(\chi\alpha^2)^2} - 2\lambda\sigma_\varepsilon^2\ln(1+\frac{(\chi\alpha)^2}{2}\sigma_\varepsilon^2)+(\chi\alpha^2\sigma_\varepsilon^2)^2\right] \tag{5.25}$$

Taking the first derivative of equation **(5.25)** with respect to λ and setting the resuling expression equal to zero yields:[74]

$$\lambda^* = \frac{\chi^2\alpha^4\sigma_\varepsilon^2\left[\chi\rho\sigma_v^2 + 2(1-\rho^2+\chi\rho\sigma_v^2)\ln(1+\frac{(\chi\alpha)^2}{2}\sigma_\varepsilon^2)\right]}{2\ln(1+\frac{(\chi\alpha)^2}{2}\sigma_\varepsilon^2)\left[\chi\rho\sigma_v^2 + (1-\rho^2+\chi\rho\sigma_v^2)\ln(1+\frac{(\chi\alpha)^2}{2}\sigma_\varepsilon^2)\right]} \tag{5.26}$$

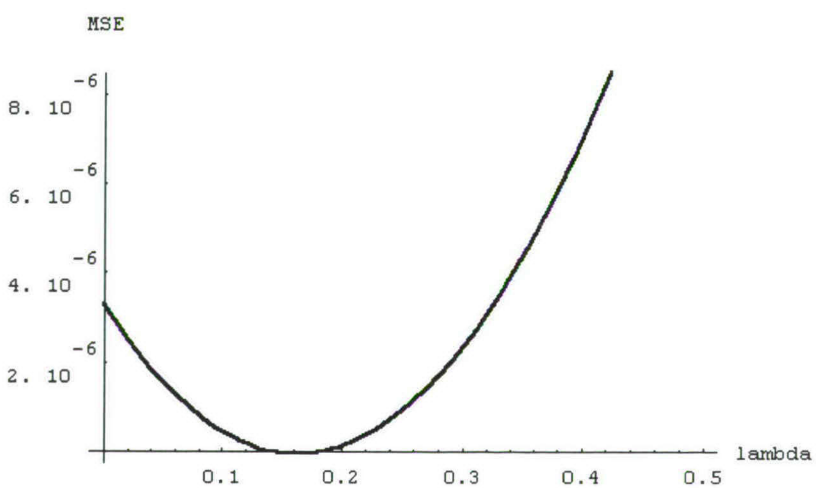
[74] Alternatively, welfare could also be improved by selecting an optimal combination of the central banker's relative weight on output stabilisation and her output target. Our intuition is that these two would be inversely related. This can be seen by adding a constant non-zero output target to the central bank's preferred value of the output gap in equation **(5.18)**. In that case it can be shown that the optimal conditional inflation forecast in equation **(5.21)** will be strictly increasing in this output target. Hence, for instance choosing an output target which exceeds potential will result in an optimal value of λ which is lower than would be the case if the output target were equal to potential output. This is because the ambitious output target *in itself* will already make the deflationary bias less severe.

Proposition 5.4:

If the expectations-augmented Phillipscurve is non-linear and if there is no information asymmetry, the central bank's optimal relative weight on output stabilisation will be strictly greater than zero ($\lambda^* > 0$).

The proof of this proposition follows immediately from equation **(5.26)**. Proposition 5.4 can be illustrated graphically by plotting the MSE of the *actual* rate of inflation (i.e. $(E(\pi^2))$) against the central bank's relative weight on output stabilisation using some plausible values of the other parameters in the model (see Figure 5.2). Later on we will use this 'baseline case' to assess the effect of several parameters on λ^* graphically. The parameter values are: α=0.3, χ=50, $\sigma_\varepsilon^2=\sigma_v^2$= 0.0004 and ρ=0.5. The first two correspond to the estimates obtained by Bean (2000) for the United Kingdom while the latter two correspond to a standard deviation of demand and supply shocks equal to 2%.

Figure 5.2: Optimal value of λ



The figure shows that given our choice of parameters the MSE of inflation $(E(\pi)^2)$ will be decreasing (and hence welfare will be increasing) in λ for relatively small values of this parameter. Hence, whereas in the linear model the appointment of a strict inflation targeter (or, equivalently, ultra-conservative central banker) was shown to be optimal, society will gain if it appoints an individual who attaches some weight to output stabilisation if the Phillipscurve is convex. This result holds even though (due to the absence of an informational asymmetry between the central banker and the public) the central banker cannot stabilise output *at all* in equilbrium (i.e. even though her relative weight on output stabilisation will not influence the Mean Squared Error of output).

The intuition is that a policy of strict inflation targeting will induce a deflationary bias (see Proposition 5.3). As argued before, this bias can be made less severe by the central banker's (futile) attempts to stabilise output since these will cause the unconditional expected rate of inflation to increase. Hence, moving from a situation of strict inflation targeting to a relatively small degree of flexible inflation targeting will reduce the deflationary bias and will therefore increase social welfare.

Finally, we can investigate the effect of changes in several parameters on λ^* by plotting the expression obtained for MSE in equation **(5.25)** against λ and each of these parameters separately. Implicitly, the relationship between the optimal level of λ and the parameter considered is then given by the line which traces out the minimum value of MSE in the three-dimensional plane. These Figures are displayed in Appendix D. First of all these figures show that the effect of changes in the variance of demand shocks (σ_ε^2) and the variance of the innovation in supply shocks (σ_v^2) on λ^* is very small. Next, for the baseline case there seems to be a clear positive relationship between the average slope of the Phillipscurve (α) and the extent to which the central bank should seek to stabilise output. This confirms the intuition obtained in Section 5.2. The same holds for the persistence of cost-push shocks (ρ). Finally, the parameter which indexes the curvature of the Phillipscurve (χ) seems to be negatively related to the optimal degree of output stabilisation.

5.4: Summary and Conclusion

In this chapter we analyse the optimal degree of conservatism in a world of flexible goods market prices where the economy's aggregate supply relationship is given by the expectations-augmented (Lucas-type) Phillipscurve. In particular, our main question is to ascertain whether or not a convex Phillipscurve of this type would entail an additional return to output stabilisation which is not present in a linear world. Obviously, such an additional gain is present if price setting is purely backward-looking since then any increases in output above long-run potential will have to be matched by more severe decreases in output to disinflate the economy. Hence, under an accelerationsist convex Phillipscurve there will be an inverse relationship between the mean and the variance of output (see Clark et al. (1995)). However, such an additional gain from output stabilisation is not immediately obvious within a Lucas-type transmission mechanism since the need to decrease output to disinflate the economy will critically depend on the central bank's credibility in that case.

As a benchmark we first analyse a linear model in which the central bank tries to stabilise inflation around the assigned target and to stabilise output around potential. Hence, we abstract from the type of Barro-Gordon (1983) credibility problem which leads to a systematic inflationary bias. However, on the assumption that cost-push shocks are persistent over time there will another type of credibility problem in monetary policy. This is because the public can partially predict the realisation of the cost-push shock. As a result, given that there is no uncertainty about central bank preferences, the central bank's reaction to this predictable part will be incorporated into inflationary expectations. This will lead to a suboptimally high variability of inflation, which could be avoided if the central bank were able to commit to not reacting to the predictable part of the cost-push shock. In the absence of a credible commitment mechanism social welfare can improved by appointing a central banker whose relative weight on output stabilisation is lower than society's relative weight. Moreover, if the central bank does not have private information and hence cannot affect output at all, it will be optimal to appoint an ultra-conservative central banker who only cares about inflation stabilisation.

Next, we extend the symmetric information model with persistent cost-push shocks by assuming that the Phillipscurve is convex. In this respect, we find that a policy of strict inflation targeting (which was shown to be optimal in the linear case) will lead to a systematic deflationary bias. Essentially, this is because the risks surrounding the conditional inflation forecast are asymmetric. We also show that, in the case of flexible inflation targeting, the unconditional expectation of inflation will be strictly increasing in the central bank's relative weight on output stabilisation. This effect arises because of the credibility problem discussed earlier. In particular, the central bank tries to stabilise output around potential and will therefore try to offset the effect of uncertainty about demand shocks and the effect of the predictable part of cost-push shocks on output. In the long run these attempts will on average cause a *ceteris paribus* increase in the expected rate of inflation since policy is always correctly and fully anticipated by the public. As for demand uncertainty the positive relation between the relative weight on output stabilisation and the unconditional expected rate of inflation arises because demand uncertainty in itself will cause output to systematically fall below potential. Furthermore, since the Phillipscurve is convex, cost-push shocks which are distributed symmetrically around zero will be translated into decreases in output which will on average be larger than the increases in output.

In a way this credibility problem can be used to improve social welfare compared to the case of strict inflation targeting. In other words, in this model there is an additional return to output stabilisation. This does not arise because it reduces the variance and increases the mean of output but because a flexible inflation targeter will induce a less severe deflationary bias than an ultra conservative central banker.

Appendix A: Proof of Proposition 5.1

Plugging the expressions obtained in equation **(5.12)** into the social loss function **(5.13)**, taking the first-order condition of the resulting expression with respect to λ and rearranging we obtain:

$$\frac{\partial E(L)}{\partial \lambda} = \frac{(\lambda-\zeta)\alpha^2}{(\alpha^2+\lambda)^3}+\frac{\lambda\rho^2}{\alpha^4(1-\rho^2)} = 0 \quad \Leftrightarrow$$

(B.1)

$$\lambda = \Phi(\lambda) \equiv \frac{\zeta\alpha^6(1-\rho^2)}{\alpha^6(1-\rho^2)+\rho^2(\alpha^2+\lambda)^3}$$

The function Φ(λ) implicitly defines λ*. In particular, Φ (λ) has the following properties:

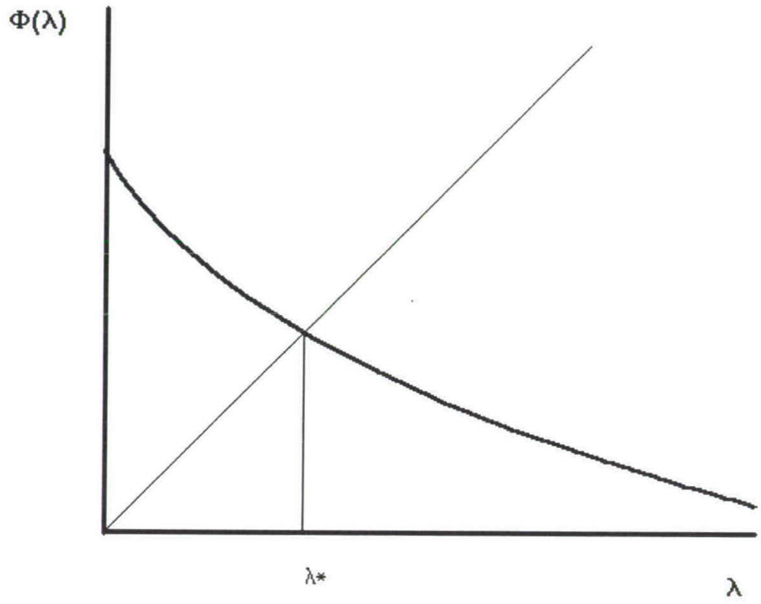
$$\Phi(0) = \zeta(1-\rho^2)$$

$$\frac{\partial\Phi(\lambda)}{\partial\lambda} = \frac{-3\zeta\alpha^6(\alpha^2+\lambda)^2\rho^2(1-\rho^2)}{(\rho^2(\alpha^2+\lambda)^3+\alpha^6(1-\rho^2))^2} < 0 \qquad \textbf{(B.2)}$$

$$\lim_{\lambda\to\infty} \Phi(\lambda) = 0$$

Using this equation we can determine λ* graphically in Figure 5.3 where the downward sloping line represents the function Φ(λ):

Figure 5.3: The optimal value of λ



From this figure we can see that proving that $\lambda^* < \xi$ amounts to proving that $\Phi(\xi) < \xi$:

$$\frac{\xi\alpha^6(1-\rho^2)}{\alpha^6(1-\rho^2)+\rho^2(\alpha^2+\xi)^3} < \xi \quad \Leftrightarrow \quad \rho^2(\alpha^2+\xi)^3 > 0 \tag{B.3}$$

Q.E.D.

Finally, by computing the following partial derivatives of $\Phi(\lambda)$ we can determine how the other parameters in the model affect λ^*:[75]

[75] The parameters ρ and ξ will also influence F(0). However, since it holds that $\partial F(0)/\partial\rho < 0$ and $\partial F(0)/\partial\xi > 0$, these effects do not change the conclusion reached in Proposition 5.2.

$$\frac{\partial\Phi(\lambda)}{\partial\alpha} = \frac{6\xi\alpha^5\lambda(\alpha^2+\lambda)^2\rho^2(1-\rho^2)}{(\alpha^6+3\alpha^4\lambda\rho^2+3\alpha^2\lambda^2\rho^2+\lambda^3\rho^2)^2} > 0$$

$$\frac{\partial\Phi(\lambda)}{\partial\rho} = \frac{-2\xi\rho\alpha^6(\alpha^2+\lambda)^3}{(\alpha^6+3\alpha^4\lambda\rho^2+3\alpha^2\lambda^2\rho^2+\lambda^3\rho^2)^2} < 0 \quad \textbf{(B.4)}$$

$$\frac{\partial\Phi(\lambda)}{\partial\xi} = \frac{\alpha^6(1-\rho^2)}{\alpha^6(1-\rho^2)+\rho^2(\alpha^2+\lambda)^3} > 0$$

Appendix B: Solving the FOC under a Non-Linear Phillipscurve

Solving out for the expectations operator across equation **(5.16)** yields:

$$\alpha E_{t-1}\pi_t\, E_{t-1}(e^{\chi\alpha y_t}) + Cov_{t-1}\left[\pi_t, \alpha e^{\chi\alpha y_t}\right] + \lambda E_{t-1}y_t = 0 \quad \textbf{(B.1)}$$

As far as the first expression on the LHS is concerned we use the following second order Taylor-expansion:

$$E_{t-1}(e^{\chi\alpha y_t}) = e^{\chi\alpha E_{t-1}y_t}(1+\frac{(\chi\alpha)^2}{2}\sigma_\varepsilon^2) \quad \textbf{(B.2)}$$

The second expression on the LHS of equation **(B.1)** can be approximated as follows:

$$Cov_{t-1}\left[\pi_t, \alpha e^{\chi\alpha y_t}\right] = \frac{\alpha}{\chi}Var_{t-1}\left[e^{\chi\alpha y_t}\right] \approx \chi\alpha^3\sigma_\varepsilon^2 \quad \textbf{(B.3)}$$

Here we have used the fact that $e^{\chi\alpha y} \approx 1+\chi\alpha y$. Plugging equations **(B.2)** and **(B.3)** back into equation **(B.1)** and rearranging we obtain equation **(5.17)**.

Appendix C: The Equilibrium Conditional Expectation of Output in the Non-Linear Model

Taking rational expectations on both sides of the Phillipscurve equation **(5.1)** (where $f(y_t)$ is determined by equation **(5.14)**) yields:

$$\frac{E_{t-1}\left(e^{\chi\alpha y_t}\right) - 1}{\chi} + \rho\mu_{t-1} = 0 \qquad \textbf{(C.1)}$$

Using equation **(B.2)** to find an expression for the first term on the LHS and rearranging we obtain:

$$e^{\chi\alpha E_{t-1}y_t} = \frac{1-\chi\rho\mu_{t-1}}{1+\frac{(\chi\alpha)^2}{2}\sigma_\varepsilon^2} \qquad \textbf{(C.2)}$$

Taking natural logs on both sides of this equation will yield equation **(5.18)** in the main text.

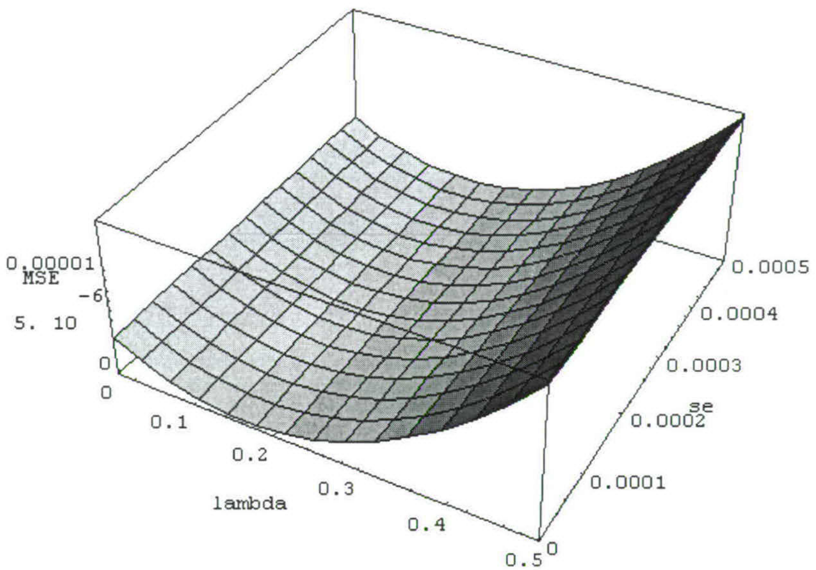
Next, equation **(C.1)** also pins down the value of the expected inflation-output trade-off since the latter is equal to:

$$E_{t-1}\gamma_t = \alpha E_{t-1}(e^{\chi\alpha y_t}) \qquad \textbf{(C.4)}$$

Substituting equation **(C.1)** into this equation yields equation **(5.19)** in the main text.

Appendix D: The effect of Several Parameters on λ^* in the Non-Linear Model

Figure 5.4: Relationship between optimal value of λ and σ_ε^2



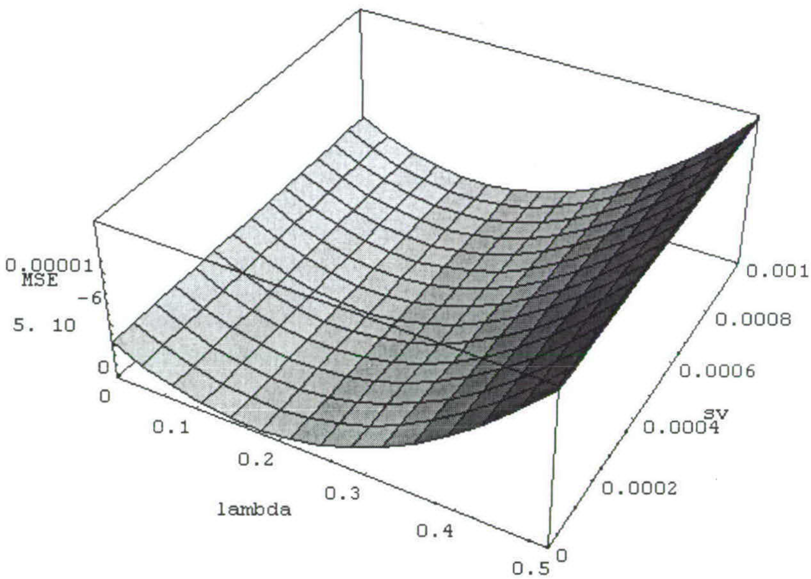


Figure 5.5: Relationship between optimal λ and $\sigma_v{}^2$

Figure 5.6: Relationship between optimal value of λ and χ

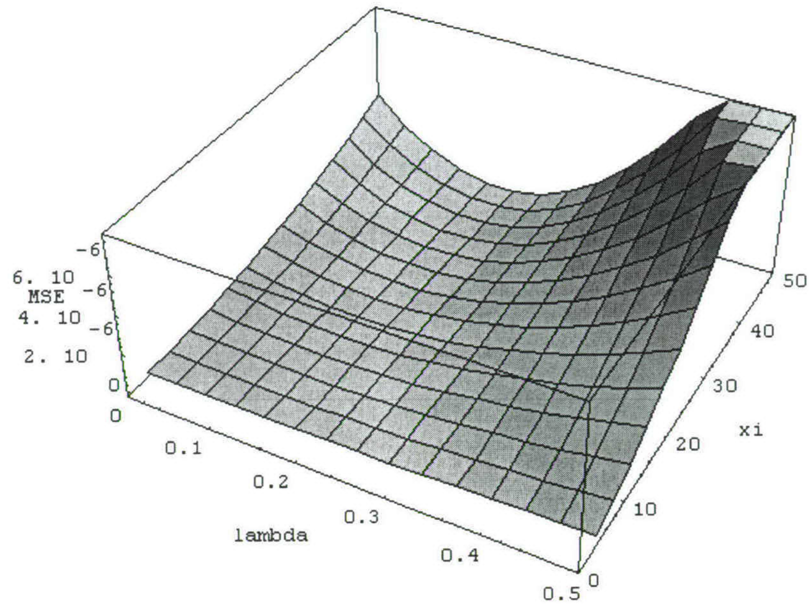


Figure 5.7: Relationship between optimal value of λ and α

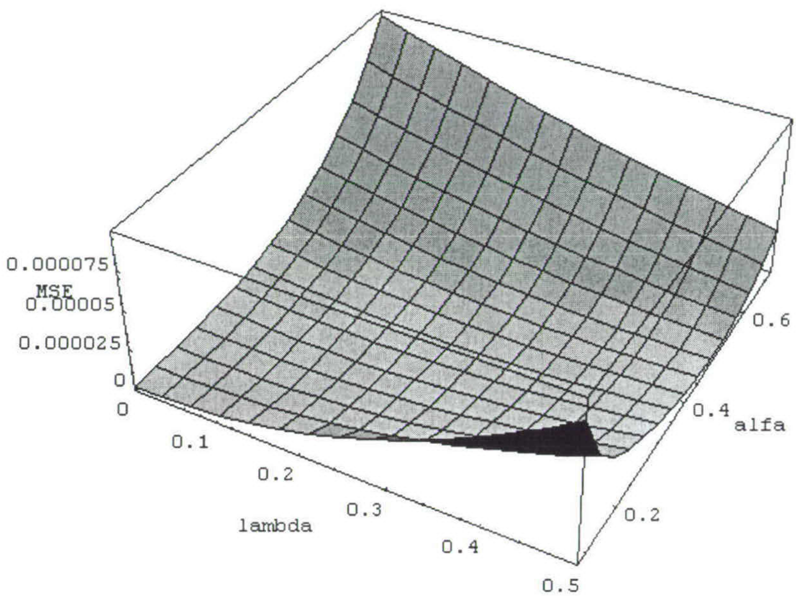
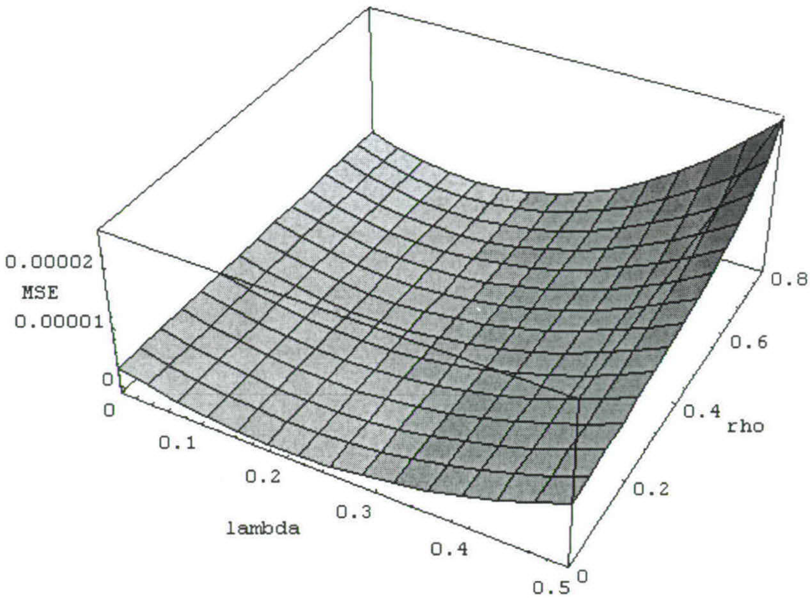


Figure 5.8: Relationship between optimal value of λ and ρ



Part III: Sterilised Foreign Exchange Interventions in an Inflation Targeting Regime

It is often argued that the approach of a large economy towards nominal exchange rate developments can be described as one of 'benign neglect'. In a world where capital can flow freely across national borders the authorities either have to choose to use the instrument of monetary policy to stabilise domestic objectives or to use it to stabilise the external value of the currency. However, if they choose to target domestic objectives this does not mean that monetary policy will not react to exchange rate movements. This is because the latter have a direct effect on the consumer price index (since it also includes imported goods) and an indirect effect via the real exchange rate which in turn affects the relative demand for home-produced goods. In other words, if for instance the central banker targets domestic objectives, a large and persistent depreciation of the nominal exchange rate will eventually lead to a rise in the domestic interest rate. However, this does not come about because of an exchange rate objective but rather because such a depreciation induces a risk of higher futuer inflation. In this respect, the relatively small effect of the nominal exchange rate on monetary policy in areas such as the US and the Euro-area should be ascribed to the fact that the share of imports and exports in GDP is relatively small. Hence, the fact that the previous chapters dealt with a closed economy can be justified by arguing that this may serve as a relatively close approximation for a large and relatively closed economic area provided the volatility of exchange rate movements is relatively small.[76]

Nevertheless, since the break-up of the Bretton-Woods system in the early 1970's the world has witnessed considerable misalignments in the exchange rates of major currencies. These large and often persistent movements of an exchange rate away from its estimated fundamental value do worry central bankers. Often, their first line of defence is the employment of foreign exchange market interventions. In principle these interventions can effect the exchange rate in three different ways.

[76]Of course, analytical convenience also serves as an important justification.

First of all, there will be an effect on the interbank money market if the central bank buys or sells foreign reserves. The resulting change in the interbank interest rate will, for a given value of the expected future exchange rate, cause a change in the nominal exchange rate. However, most of the time central banks choose to sterilise this effect by conducting an opposite transaction using domestic government securities.

Secondly, a foreign exchange intervention may be effective because of the portfolio balance effect. This effect will only arise if investors do not consider the two currencies in question to be perfect substitutes, i.e. if uncovered interest parity does not hold because of which investors will demand a risk premium. However, even if currencies are not perfect substitutes it is unlikely that the intervention will have much effect since the intervention volume is generally very small compared to the daily turnover on the foreign exchange markets.

Finally, foreign exchange interventions may directly alter the market's expectation of the future exchange rate through the signalling effect. For instance, the central bank may use foreign exchange interventions to 'announce' a future change in monetary policy. Alternatively, the central bank may 'put its money where its mouth is' by backing up statements that it feels that the exchange rate is out of line with its fundamental value. As documented by Dominguez and Frankel (1993) such interventions are most likely to be effective when there are clear indications that the current exchange rate cannot be supported by economic fundamentals and when the intervention is supported by all the central banks involved. When the market doubts whether or not a sustained appreciation or depreciation of a particular currency can be maintained for much longer, a coordinated intervention may 'prick the bubble' and cause the market sentiment to turn around. Given the importance of the signalling channel with respect to the effectiveness of foreign exchange interventions the central bank must have an information advantage over market participants.

The purpose of Chapter 6 is to study the effect of this information advantage which comes about because of two reasons. First of all, the central bank has private information about its exchange rate target and, secondly, it provides a noisy signal of the actual intervention volume. We study a repeated game between the central bank and speculators in which the latter use reported intervention volumes to learn about the central bank's exchange rate target over time. This allows use to determine the effect of several parameters on the intervention bias (i.e. the amount of foreign exchange the central bank buys or sells without this having any effect on the exchange rate). We also determine the effect of ambiguity on the covariance between exchange rate movements and the central bank's target and on the variance of exchange rate movements.

Chapter 6: The Advantage of Hiding Both Hands: Foreign Exchange Intervention, Ambiguity and Private Information

6.1: Introduction

Since the demise of the Bretton Woods system, sterilized foreign exchange interventions are regarded as the main exchange rate policy tool for a large industrial country which chooses to focus its monetary policy stance almost exclusively on domestic objectives. As documented by Almekinders and Eijffinger (1991) and Edison (1993) these interventions (which by definition do not affect the interbank money market) may derive their effectiveness from two sources. First of all, provided otherwise identical domestic and foreign assets are imperfect substitutes, the exchange rate may be affected via the *portfolio balance channel*. It is not likely, however, that central banks can induce a significant imbalance in investors' portfolios since the amount of official reserves is dwarfed by the daily turnover in the foreign exchange markets[77]. Hence, if the central bank is to have any hope of pursuing an independent exchange rate target it will have to rely exclusively on the *signalling* or *expectations channel*. The idea behind this is that sterilized interventions can have a direct impact on exchange rate expectations if they transmit hitherto privately held information to the market.

One approach to study the attempts on the part of the central bank to exploit the possible effectiveness of this channel is to model intervention policy as a game between speculators, on the one hand, and the central bank, on the other. In this respect, Almekinders (1995,1996) has developed a static exchange rate policy game of symmetric information in which the central bank's attempts to exploit the signalling channel will always be futile. The basic reason for this is that the central bank has no private information because of which interventions will not provide the market with information it did not have beforehand. A certain degree of *policy*

[77] It should be mentioned that the evidence on this presumed ineffectiveness is not clear cut. Dominguez and Frankel (1993b) have provided new evidence on the statistical significance of the portfolio balance effect. Nevertheless, because this paper focusses on the signalling channel we will assume the portfolio balance channel to be completely ineffective.

secrecy[78] thus seems to be crucial in rendering effectiveness to sterilized interventions. This observation is central to the model constructed by Bhattacharya and Weller (1992) who interpret this policy secrecy as private information about the central bank's (short-term) exchange rate target. The aim of this paper is obtain a better understanding of observed intervention behavior by extending the theoretical insights of the afore-mentioned papers. To this end we decompose policy secrecy into asymmetric information concerning the central bank's objectives, on the one hand, and *ambiguity* about its actions, on the other. Moreover, we also allow for reputational effects by studying a dynamic game.

The remainder of this paper is organized as follows. Section 2 will outline the model. Subsequently we present the dynamically consistent solution in Section 3. Section 4 will assess the effect of asymmetric information on the equilibrium intervention volume in general, and on the intervention bias, in particular. Furthermore, we will provide an investigation of the political and institutional factors which determine the size of this bias. Next, Section 5 will investigate the effect of ambiguity in the context of intervention policy. Finally, Section 6 will summarize our main conclusions.

6.2: The Exchange Rate Policy Game

In this section we will extend the static exchange rate policy game developed by Almekinders (1995,1996). It is a well-established fact that structural models fail to explain short run exchange rate movements (for a survey see MacDonald and Taylor (1992)). For instance, Meese and Rogoff (1983) found that a simple random walk outperforms these models for forecast horizons up to one year. In the light of this evidence we assume that the spot rate is (s_t)[79] is determined by the following equation:

$$\Delta s_t = \alpha + \delta(INV_t^R - INV_t^e) + \varepsilon_t \qquad with \qquad \varepsilon_t = \zeta\varepsilon_{t-1} + \mu_t \quad ; \quad 0 < \zeta < 1 \quad \mu_t \sim N(0,\sigma_\mu^2) \tag{6.1}$$

[78] The practical relevance of policy secrecy is extensively documented in Goodfriend (1986)

[79] s_t is defined as the amount of domestic currency per unit of the foreign currency.

The parameter α can be interpreted as the underlying exchange rate trend which is based on long run fundamentals (e.g. the PPP-implied trend). Secondly, exchange rate returns are assumed to be subject to a stochastic (demand) shock (ε_t) which results from the interaction between various exchange market participants and their reaction to all sorts of 'news' which hits the market continuously. To allow for the possibility of bandwagon effects (for instance caused by the widespread use of technical analysis, see Taylor and Allen (1992) and Goodhart (1988)) we assume that this shock follows an AR(1) process where the parameter ζ denotes the degree of exchange rate persistence. Finally, the second term on the RHS captures the effect of 'news' about intervention operations. In other words, this term represents the signalling channel of intervention. It will be shown that intervention operations are driven by the central bank's preferred rate of depreciation which is unknown to the market. Consequently, upon observing a discrepancy between the *reported* intervention volume (INV_t^R), on the one hand, and the *expected* intervention volume (INV_t^e), on the other, the market will receive new information about the central bank's preferences in this respect. Analogous to the effect of other sorts of 'news' this will lead agents to revise their forecast for future exchange rates.

The intervention volume *reported* to the foreign exchange markets via the financial press and Reuters screens (INV_t^R) will typically not be equal to the *actual* intervention volume (INV_t). This is because central banks never reveal the exact magnitude of the latter. On the other hand, as documented by Dominguez and Frankel (1993a), their presence in the foreign exchange market rarely goes unnoticed. We model this by assuming that the market's perception of the actual intervention volume is subject to a random control error (η_t) :

$$INV_t^R = INV_t + \eta_t \qquad with \qquad \eta_t \sim N(0,\sigma_\eta^2) \tag{6.2}$$

The central bank is assumed to minimise an intertemporal loss function (Λ^{CB}) which consists of the discounted value of expected 'period' loss functions (L_t^{CB}):

$$\Lambda^{CB} = E_0 \left[\sum_{t=0}^{\infty} \beta^t L_t^{CB} \right] \quad with \quad 0 \leq \beta \leq 1$$

$$L_t^{CB} = \frac{1}{2}(kINV_t)^2 + \frac{\varphi}{2}(\Delta s_t - T_t)^2 \qquad \textbf{(6.3)}$$

The parameter β denotes the central bank's discount factor while φ denotes the central bank's relative weight on exchange rate stabilisation. The presumption is that there will be a loss whenever the change in the spot rate (Δs_t) differs from the central bank's target (T_t). Undesired exchange rate movements can potentially be mitigated by means of sterilized interventions (INV_t)[80] which induce a cost of k per unit of foreign exchange traded. This cost can be explained by the bureaucratic costs involved and the fact that the central bank may incur a loss on its purchases (sales) of foreign exchange if these turn out to be unsuccessful in preventing the domestic currency from appreciating (depreciating). It can be argued that these costs may be offset by potential gains. However, a risk averse central banker will be inclined to limit the amount of reserves to be put at stake.

The central bank's short-term target consists of the long run exchange rate trend α and a stochastic shock p_t which reflects the central bank's private information. The latter follows an AR(1) process where ρ denotes the degree of target persistence:

$$T_t = \alpha + p_t \quad with \quad p_t = \rho p_{t-1} + \nu_t \;, \quad 0 < \rho < 1 \quad and \quad \nu_t \sim N(0,\sigma_\nu^2) \qquad \textbf{(6.4)}$$

In traditional signalling models (Mussa (1981)) there is a strong and direct relationship between sterilized interventions and future changes in the money stock. However, in our view, this explanation is inconsistent with the fact that interventions are meant to be an *independent* policy tool. Clearly, this does not suggest that there should be any *systematic* relationship between sterilized interventions and subsequent changes in the money supply. By contrast, Kenen (1987) has suggested that interventions can be used to transmit the central bank's view

[80] A positive (negative) value of INV_t denotes a purchase (sale) of foreign exchange by the central bank.

on exchange rate developments to the markets[81]. The presumption in our model is that thus that p_t reflects the central bank's changing assessment of what it considers to be the appropriate rate of depreciation. While this assessment will undoubtedly contain private information about future fundamentals, it should be stressed that this is markedly different from stating that the central bank will systematically change the money stock every time it intervenes.

Finally, the *sequence of events* in the stage game can be summarised as follows:

Figure 6.1: Sequence of events

1	2	3	4	5	6
ε_t realizes	speculators set INV_t^e	p_t realizes	CB sets INV_t	η_t realizes, subsequently INV_t^R is revealed	Δs_t is realized

6.3: The Dynamically Consistent Solution

To simplify the calculations it will be assumed that speculators have a perfect observation on the state of the central bank's target realized two periods earlier (p_{t-2}). This means that the current intervention volume (INV_t) will only influence the expected volume in the next period (INV_{t+1}^e). In reality the learning process involved will probably extend to more than one period and will fade out gradually (in the sense the public will place a higher weight on more recent periods relative to less recent periods). Nevertheless, the main implications of asymmetric information and the essence of the learning process can also be demonstrated by means of a

[81] In this respect Kenen notes that *'...Intervention can be used for underscoring the authorities' commitment to current policies or for trying to persuade the market that the prevailing exchange rate is inconsistent with the fundamentals...(it) can also be used to change the market's confidence in its own projections...'* (Kenen 1987,p. 198)

short-lived information advantage. The equilibrium concept[82] we will use is of the Nash-variety and can be formulated as follows:

* In every period the central bank selects the intervention volume so as to minimize its intertemporal loss function (Λ^{CB}) given the exchange rate constraint **(6.1)** and given its perception of the market's expectations formation process.
* Given their perception of the policy rule followed by the central bank and the information currently available, speculators form their expectations about the intervention volume so as to minimize the conditional mean squared forecast error ($E[\ (INV_t - INV_t^e)^2 \mid I_t\ ]$) in each period[83].
* The policy rule as perceived by speculators is identical to the policy rule that comes about in equilibrium and, conversely, the perceptions of the central bank concerning the expectations formation process is identical to the actual process used by speculators in equilibrium.

Hence, the actual volume of interventions (INV_t) and the expectation of this volume (INV^e_t) will be determined simultaneously. When substituting equations **(6.1), (6.2)** and **(6.4)** into the expression for L_t^{CB} in equation **(6.3)**, it can be seen that INV_t will depend upon ε_t , p_t , INV_t^e and, because of the link between periods, also on $E_t(p_{t+1})$, $E_t(INV_{t+1}{}^R\text{-}INV^e_{t+1})$ and $E_t(\varepsilon_{t+1})$[84]. Since the loss function of the central bank is *quadratic* in both terms we will postulate the following *linear* intervention reaction function in which D_i , i=1,..,6 are the coefficients to be determined:

---

[82] The solution concept used is that of *dynamic consistency* which is weaker than the concept of subgame perfection (see Cukierman (1992), Chapter 11). Dynamic consistency requires that the player's actions be optimal at each point in time along the equilibrium path only while subgame perfection (or its equivalent in games of incomplete information) puts requirements on beliefs and actions off the equilibrium path as well.

[83] This 'loss function' can be justified on the grounds that, by assumption, strategic interaction between the central bank and the speculators will take place *after* the realisation of ε_t. Hence, the model allows speculators to be heterogeneous with repect to their access to and reaction to 'news' in general. It is *only* for the purpose of studying the reaction of speculators to 'news' about the central bank's objectives that we implicitly impose the condition that all speculators have the same information. This, in turn, allows us to analyse the model as a game between the central bank, on the one hand, and a representative speculator, on the other.

[84] Throughout this paper we will use the following convention: expectations conditioned on the central bank's information set in time t will be denoted as $E_t(...)$ while expectations conditioned on the information set of the speculators in time t will be denoted as $E(...|I_t)$.

$$INV_t = D_1\varepsilon_t + D_2p_t + D_3INV_t^e + D_4E_t(p_{t+1}) + D_5E_t(INV_{t+1}^R - INV_{t+1}^e) + D_6E_t(\varepsilon_{t+1}) \quad \textbf{(6.5)}$$

These coefficients can be determined by writing out the central bank's intertemporal loss function (Λ^{CB}) for periods t and t+1, taking expectations conditional on the central bank's information set in period t. Computing the first-order condition of the resulting equation and using the expression obtained for $\partial INV_{t+1}^e/\partial INV_t$ in Appendix A, we arrive at an equation which has the same form as equation **(6.5).** Equating coefficients across these two equations yields the following:

$$\begin{aligned}
D_1 &= -\frac{\varphi\delta}{(k^2+\varphi\delta^2)} \\
D_2 &= \frac{\varphi\delta}{(k^2+\varphi\delta^2)} = -D_1 \\
D_3 &= \frac{\varphi\delta^2}{(k^2+\varphi\delta^2)} \\
D_4 &= -\frac{\beta\varphi\delta\rho\theta}{k^2} \\
D_5 &= \frac{\beta\varphi\delta^2\rho\theta}{k^2} = -\delta D_4 \\
D_6 &= \frac{\beta\varphi\delta\rho\theta}{k^2} = -D_4
\end{aligned} \quad \textbf{(6.6)}$$

The model thus yields explicit solutions for D_1 , D_2 and D_3. Through the dependence of θ (which is defined in Appendix A) on the coefficient D_4 it also yields an implicit solution for D_4 and, thereby, also for D_5 and D_6. Using the results obtained in Appendix A, this implicit solution can be written as follows:

$$D_4 = -\frac{\beta\varphi\delta\rho[\varphi\delta/(k^2+\varphi\delta^2) + \rho D_4]^2\sigma_v^2}{k^2([\varphi\delta/(k^2+\varphi\delta^2) + \rho D_4]^2\sigma_v^2 + \sigma_\eta^2)} \equiv F(D_4) \quad \textbf{(6.7)}$$

In Appendix B it is shown that there always exists at least one solution for D_4 for which it holds that:

$$- \frac{1}{\rho} \frac{\varphi\delta}{(k^2+\varphi\delta^2)} < D_4 < 0 \tag{6.8}$$

Using equations **(6.5)** and **(6.6)** and observing that $E_t(\varepsilon_{t+1}) = \zeta\varepsilon_t$ and $E_t(p_{t+1}) = \rho p_t$, we can now derive the following expression for the central bank's reaction function[85]:

$$INV_t = - \frac{\varphi\delta}{(k^2+\varphi\delta^2)} [\varepsilon_t - p_t - \delta INV_t^e] - D_4 [\zeta\varepsilon_t - \rho p_t + \delta E_t(INV_{t+1} - INV_{t+1}^e)] \tag{6.9}$$

The economic interpretation of this equation is quite intuitive. The first term between brackets represents the current undesired depreciation of the spot rate (Δs_t-T_t) under the condition that the central bank abstains from interventions. Provided this term is strictly positive the 'period' loss which results from this can be mitigated by *selling* foreign exchange ($INV_t < 0$). However, as a side effect this will lead the market to expect the central bank to buy less foreign exchange in the next period as well ($\partial INV_{t+1}^e/\partial INV_t > 0$). The second term between brackets denotes the expected undesired depreciation in the next period conditional on the information set in period t ($E_t(\Delta s_{t+1}$-T_{t+1})). On the assumption that the this term is strictly positive as well the central bank faces a ceteris paribus incentive to *buy* foreign exchange ($INV_t > 0$). After all, the concomitant *increase* in next period's expected intervention volume will lower next period's expected loss since it will give the central bank more scope to counter the expected undesired depreciation by means of surprise interventions. Hence, the central bank is basically faced with an *intertemporal trade-off*. Lowering the loss caused by undesired exchange rate movements in the current period will make it harder to mitigate the expected loss in the next period.

Having obtained the central banker's intervention reaction function we now derive the speculators' reaction function. In Appendix C it is shown that the latter is given by:

---

[85] Since $E_t(\eta_{t+1}) = 0$, the expression $E_t(INV_{t+1}^R - INV_{t+1}^e)$ has been replaced by $E_t(INV_{t+1} - INV_{t+1}^e)$.

$$INV_t^e = -[\frac{\varphi\delta+\zeta D_4(k^2+\varphi\delta^2)}{k^2}]\,\varepsilon_t + \\ +[\frac{\varphi\delta+\rho D_4(k^2+\varphi\delta^2)}{k^2}]\,[\rho^2 p_{t-2} + \rho\theta(\nu_{t-1} + \frac{(k^2+\varphi\delta^2)}{\varphi\delta+\rho D_4(k^2+\varphi\delta^2)}\,\eta_{t-1})] \tag{6.10}$$

This equation shows that the market essentially reacts to the exchange rate shock (ε_t) on which it has the same information as the central bank, on the one hand, and to its *perception* of the current state of the central bank's target (p_t), on the other. The latter can be broken down into the innovation realised two periods earlier (p_{t-2}) which is contained in the market's information set and an optimal prediction of the innovation to the target realised in the previous period (ν_{t-1}).

6.4: The Impact of Asymmetric Information on the Equilibrium Intervention Volume and the Intervention Bias

Next, we can calculate the equilibrium volume of intervention by inserting equation **(6.10)** into **(6.9)** and using the expression obtained for $E_t(INV_{t+1}-INV_{t+1}^e)$ in Appendix D:

$$INV_t = -[\frac{\varphi\delta+\zeta D_4(k^2+\varphi\delta^2)}{k^2}]\varepsilon_t + [\frac{\varphi\delta+\rho D_4(k^2+\varphi\delta^2)}{k^2}]\rho^2 p_{t-2} + \\ [\frac{\varphi\delta}{(k^2+\varphi\delta^2)}+\rho D_4][1+\theta\frac{\varphi\delta^2}{k^2}]\rho\nu_{t-1} + [\frac{\varphi\delta}{(k^2+\varphi\delta^2)}+\rho D_4][1-\delta\rho D_4(1-\theta)]\nu_t + \frac{\varphi\delta^2}{k^2}\rho\theta\eta_t \tag{6.11}$$

To interpret this equation it turns out to be instructive to compare it with the equilibrium intervention volume which will result in the presence of *symmetric* information[86]. In that case the current stance of the central bank's short-term target (p_t) will be contained in the market's information set. Consequently, the game will be completely transparent for both players and

[86] Symmetric information should be clearly distinguished from a case in which the central bank announces its exchange rate target. As argued by Stein (1989), given that the public believes these announcements, the central bank faces an incentive to cheat because of which the central bank cannot make any *precise* and credible announcements on its exchange rate target. In the case of symmetric information the central bank is completely transparant to the public because of which it does not *need* to make any announcements.

reported intervention volume will always be equal to the *actual* volume:

$$INV_t^R = INV_t \quad \forall\, t \quad \Leftrightarrow \quad \sigma_\eta^2 = 0 \tag{6.12}$$

Furthermore, since speculators no longer need past intervention volumes to predict the current state of the target, the game will simplify into a string of unrelated one-period problems[87]. The central bank's reaction function can then be obtained by plugging **(6.1)** and **(6.4)** into the 'period' loss function **(6.3).** Taking the first order condition of the resulting expression and rearranging yields the following:

$$INV_t = -\frac{\varphi\delta}{(k^2+\varphi\delta^2)}[\varepsilon_t - p_t - \delta INV_t^e] \tag{6.13}$$

Taking rational expectations across this equation and using the fact that $p_t = \rho^2 p_{t-2} + \rho\nu_{t-1} + \nu_t$, we obtain the following expression for the (Nash) equilibrium intervention volume under symmetric information:

$$INV_t = INV_t^e = -\frac{\varphi\delta}{k^2}\varepsilon_t + \frac{\varphi\delta}{k^2}[\rho^2 p_{t-2} + \rho\nu_{t-1} + \nu_t] \tag{6.14}$$

When comparing the expressions obtained for the equilibrium intervention volume under asymmetric and symmetric information (equations **(6.11)** and **(6.14)** respectively) we can derive the following proposition about the central bank's *degree of policy activism*:

Proposition 6.1:

> The absolute value of the reaction coefficients for ε_t , p_{t-2} , ν_{t-1} and ν_t will be *strictly less* if the central bank retains *private information* about the short-term exchange rate target compared to the situation where the central bank chooses to reveal the target perfectly.

Proof: see Appendix E

[87] i.e. $\partial INV_{t+i}^e / \partial INV_t = 0 \;\; \forall\, i \geq 0$.

The intuition is that the central bank cannot manipulate future intervention expectations in the absence of private information. This means that these expectations will not exert a deterrent effect on the current intervention volume. Hence, the central bank has good reasons to retain private information since this provides an instrument to mitigate the time-inconsistency problem in intervention policy. As pointed out by Almekinders (1995, 1996), in a symmetric world it would be optimal if the central bank were able to make a commitment not to intervene at all. However, since the central bank cannot make a credible commitment, it will end up in the Nash equilibrium described by equation **(6.14).** Not revealing the short-term target will both reduce the degree of central bank activism and render effectiveness to intervention policy because of which the central bank will generally be better off than in a symmetric world.

6.4.1 The intervention bias under asymmetric information

Nevertheless, in general the central bank will still be faced with a time-inconsistency problem since its reaction to the exchange rate shock (ε_t) will always be fully anticipated[88]. To analyse the impact of various political and economic parameters on the severity of the time-inconsistency problem, we will define the *intervention bias* (B_t) as the absolute value of the central bank's reaction to ε_t:

$$B_t \equiv \left| - \left[\frac{\varphi\delta + \zeta D_4(k^2 + \varphi\delta^2)}{k^2}\right] \varepsilon_t \right| \tag{6.15}$$

From this equation we can derive the following proposition:

[88] This can clearly be seen from equation **(6.10)** and **(6.11)** since the coefficients for ε_t are exactly the same across these equations.

Proposition 6.2:

The intervention bias (B_t) at any given time will be *lower*,

1. the *longer* the planning horizon of the central bank (β),
2. the *higher* the variance of the innovation in the target (σ_v^2),
3. the *lower* the variance of the misperception error (σ_η^2),
4. the *higher* the degree of persistence in the exchange rate shock (ζ),

Proof: see Appendix F

As for the effect of the length of the policy horizon (β) this is result is rather trivial. When the central bank becomes more concerned with expected future losses, it will pay more attention to the adverse effect of its current actions on next period's expectations. An important implication of this result is that a longer policy horizon will entail an improved ability to counter the time-inconsistency problem because it decreases intervention costs without in principle sacrificing the central bank's ability to influence the exchange rate. It is well-known from the literature that the length of the policy horizon is positively related to the degree of *central bank independence* (see Cukierman (1992), Chapter 18 and Eijffinger and De Haan (1996)). In this respect, we should expect a very independent central bank, such as the Deutsche Bundesbank, to trade less foreign exchange reserves in vain than more dependent (i.e. more myopic) central banks.

Next, the effects on the variance of the innovation to the target (σ_v^2) and the variance of the market's misperception error (σ_η^2) can be understood from the way in which they affect the *speed of learning* (θ) as shown in Proposition 6.3 below. This parameter can be seen as a measure of the extent to which the central bank will transmits information about its preferences on average. As such it is an important determinant of the link between periods (as measured by $\partial INV_{t+1}^e/\partial INV_t$). An increase in the relative variance σ_v^2/σ_η^2 will strengthen this link and, therewith, the deterrent effect of next period's expectations on the current intervention volume.

Finally, a larger degree of persistence in the exchange rate shock (ζ) will increase the effect of the current exchange rate shock on next period's expected exchange rate shock. Again this will cause the central bank to be more aware of the future consequences of its current intervention policy. This result is intuitively plausible since the degree of exchange rate persistence can be seen as an indicator of the strength of the market sentiment underlying bandwagon effects. Hence, irrespective of the central bank's response to its own subjective preferences, its degree of activism towards the objectively verifiable exchange rate shock will be weaker if this underlying sentiment becomes stronger.

6.4.2 The determinants of market's speed of learning

From the preceding discussion it will be clear that the market's speed of learning (θ) plays a crucial role in this model. In Appendix A this parameter is defined as follows:

$$\theta \equiv \frac{(D_2+\rho D_4)^2\sigma_\nu^2}{(D_2+\rho D_4)^2\sigma_\nu^2 + \sigma_\eta^2} \tag{6.16}$$

In each period speculators effectively observe a linear combination of last period's misperception error (η_{t-1}) and last period's innovation to the target (ν_{t-1}). Equation **(6.16)** is a measure of the *average* fraction of this linear combination which is caused by preference shocks. Hence, an increase in θ means that the central bank will, on average, transmit more information about its preferences ex post. The effect of various institutional parameters on the speed of learning is summarized by the following proposition:

Proposition 6.3:

The market's speed of learning (θ) will be *higher*,

1. the *shorter* the central bank's planning horizon (β),
2. the *higher* the variance of the innovation to the target (σ_ν^2),
3. the *lower* the variance of the misperception error (σ_η^2),

Proof: see Appendix F

A longer planning horizon will reduce the degree of policy activism with respect to preference shocks. This will diminish the relative weight of these shocks in the afore-mentioned linear combination of preferences shocks and misperception errors observed by speculators. Hence, a more independent central bank will also be more inclined to preserve its information advantage[89]. Similarly, if preferences are relatively unstable over time and if the volatility of misperception errors is relatively small (i.e. if σ_v^2/σ_η^2 is relatively large) it will generally be easier for speculators to deduce these preferences from observed intervention operations.

6.5: The Impact of Ambiguity

Many authors (e.g. Dominguez and Frankel (1993a)) have criticized the tendency of central banks tend to keep their intervention volumes secret. However, as noted before, this tendency seems to be pretty persistent both over time and across different central banks. In this section we wil concentrate on the effects of ambiguity from a positive persective.

First of all, in Appendix G it is shown that the covariance between exchange rate movements and the central bank's target reads as follows:

$$Cov(\Delta s_t, T_t) = \delta(D_2+\rho D_4)[(1-\theta)\rho^2 + (1-\delta\rho D_4(1-\theta))]\sigma_v^2 \qquad \textbf{(6.17)}$$

This equation can be used to assess the extent to which central bank preferences will affect exchange rate movements:

[89] In this respect Dominguez and Frankel (1993a, p. 85) have noted that, '...consistently only about one-quarter of the variation in Bundesbank intervention is predictable..' .

Proposition 6.4:

Maintaining a strictly positive degree of ambiguity will increase the covariance between exchange rate movements (Δs_t) and the central bank's exchange rate target (T_t) compared to the situation where the central bank perfectly reveals the intervention volume.

Proof: see Appendix G

Although positive and negative intervention induced exchange rate movements will cancel out *on average*[90], a positive degree of ambiguity will allow the central bank to exert more control over the distribution of intervention surprises over time. This stems from the fact that the ceteris paribus effect of shocks to the central bank's preferences on both present and future exchange rate movements (i.e. the absolute value of $\partial\Delta s_t/\partial v_{t-}$ and $\partial\Delta s_t/\partial v_t$) will be strictly larger if the central bank chooses to send ambiguous signals of the actual intervention volume. The intuition is that in the absence of ambiguity the intervention volume will always perfectly reveal the short-term target to the market *ex post*. Consequently, the central bank's ability to influence the spot rate through the signalling channel will be rather limited since speculators only face uncertainty about the *current* innovation to the target (v_t). By introducing ambiguity the central bank can signal some of its private information without completely revealing the current state of its preferences at the same time.

Secondly, we can investigate the effect of ambiguity on the variance of exchange rate movements:

$$Var(\Delta s_t) = \delta^2(D_2+\rho D_4)^2[(1-\theta)^2\rho^2 + (1-\delta\rho D_4(1-\theta))^2]\sigma_v^2 + \delta^2(1+\rho^2\theta^2)\sigma_\eta^2 + \sigma_\varepsilon^2 \quad \textbf{(6.18)}$$

[90] From equation **(6.17)** it can be seen that $E(INV_t^R - INV_t^S)$, this result rests on our assumption that the central bank cannot systematically fool the markets ($E(\eta_t)=0$) and the assumption that the central bank does not possess any systematic information advantage concerning future fundamentals ($E(\mu_t)=0$).

Except for the last term on the RHS all other terms are directly related to intervention policy. From equation **(6.18)** we can derive the following proposition:

Proposition 6.5:

The introduction of ambiguity in intervention policy ($\sigma_\eta^2 > 0$) will increase the degree of intervention *uncertainty* ($E(INV_t^R - INV_t^e)^2$) and will, consequently, also increase the volatility of exchange rate movements ($Var(\Delta s_t)$).

Proof: see Appendix G

The intuition is that ambiguity will amplify the variance of preference shocks (σ_t^2) since it increases the effect of these shocks on exchange rate movements. On top of that, it will also introduce an additional degree of uncertainty via the variance of misperception errors (σ_η^2).

To sum up, it turns out that the transmission of noisy rather than clear signals may constitute an important complement of private information since it extends the information advantage enjoyed by the central bank. In spite of this, it cannot be concluded a priori that the practice of sending ambiguous signals will be in the interest of the central bank from the perspective of minimizing its intertemporal loss function. More specifically, since both the variance of exchange rate movements ($Var(\Delta s_t)$) and the covariance between these movements and the central bank's preferences ($Cov(\Delta s_t, T_t)$) will increase as a result of ambiguity, it is not a priori clear what will happen to $E(\Delta s_t - T_t)^2$. Moreover, from Proposition 6.2 it can be seen that the intervention bias will be strictly lower in the absence of ambiguity. This means that the amount of futile foreign exchange transactions will increase as a consequence of noisy signalling. However, such considerations take us away from the positive description of central bank practices towards the normative consideration whether or not central banks should choose a positive degree of ambiguity when given the choice. We leave this question for future research.

6.6: Conclusion

This paper examines the central bank's attempts to influence the spot rate by means of sterilized foreign exchange interventions. To this end we have examined a dynamic game in which the central bank retains private information about its short-term exchange rate target and in which speculators are subject to ambiguity concerning the actual intervention volume. By the very act of intervening the central bank will transmit some information about its short-term target to the market which will lead speculators to revise their expectations about the future spot rate. In this respect the effect of 'news' about the preferences of the central bank is identical to the effect of other sorts of 'news'. The dynamic game analyzed in this paper contains a learning process on the part of speculators which introduces a link between periods. This link will induce the central bank to take the future consequences of its current actions into account which will generally reduce the degree of activism. A particularly interesting feature of this paper is that provides a theoretical foundation for the effects of ambiguity. On the one hand, this practice will allow the central bank to engineer a closer relationship between exchange rate movements and its preferences. However, on the other hand, it will also increase the volatility of these exchange rate movements.

We also examine the effect of various institutional parameters on the central bank's ability to deal with the time-inconsistency problem in intervention policy. In this respect we find that the futile component of intervention operations (i.e. the intervention bias) will decrease if the central bank in question becomes more independent. This effect arises because the length of the policy horizon is positively related to the degree of central bank independence.

Appendix A: Calculation of $\partial INV_{t+1}^e/\partial INV_t$

Leading equation **(6.5)** by one period and taking expectations conditional on the market's information set in period t+1 (I_{t+1}) we obtain:

$$\begin{aligned} E(INV_{t+1}|I_{t+1}) \equiv INV_{t+1}^e = D_1E(\varepsilon_{t+1}|I_{t+1}) + D_2E(p_{t+1}|I_{t+1}) + D_3INV_{t+1}^e + \\ D_4E(E_{t+1}(p_{t+2})|I_{t+1}) + D_5E(E_{t+1}(INV_{t+2}-INV_{t+2}^e)|I_{t+1}) + D_6E(E_{t+1}(\varepsilon_{t+2})|I_{t+1}) \end{aligned} \quad \textbf{(A.1)}$$

With regard to this equation the following can be noted:

$$\begin{aligned} &E(p_{t+1}|I_{t+1}) = \rho^2 p_{t-1} + \rho E(\nu_t|I_{t+1}) \\ &E(E_{t+1}(p_{t+2})|I_{t+1}) = E(p_{t+2}|I_{t+1}) = \rho^3 p_{t-1} + \rho^2 E(\nu_t|I_{t+1}) \\ &E(E_{t+1}(\varepsilon_{t+2})|I_{t+1}) = E(\varepsilon_{t+2}|I_{t+1}) = \zeta\varepsilon_{t+1} \\ &E(E_{t+1}(INV_{t+2}-INV_{t+2}^e)|I_{t+1}) = E((INV_{t+2}-INV_{t+2}^e)|I_{t+1}) = 0 \end{aligned} \quad \textbf{(A.2)}$$

The last expression in **(A.2)** simply states that the expected value of surprise interventions in period t+2 based on the information available to speculators in period t+1 should be equal to zero. Otherwise, these interventions could not have been unexpected in the first place. Plugging equation **(A.2)** into **(A.1)** and rearranging we obtain:

$$INV_{t+1}^e = \frac{1}{1-D_3}\left[(D_1+\zeta D_6)\varepsilon_{t+1} + (D_2+\rho D_4)(\rho^2 p_{t-1} + \rho E(\nu_t|I_{t+1})\right] \quad \textbf{(A.3)}$$

To get an expression for $E(\nu_t|I_{t+1})$ we note that, using **(6.3)** and **(6.5),** speculators will have observed the following in period t:

$$\begin{aligned} INV_t^R = (D_1+\zeta D_6)\varepsilon_t + D_2(\rho p_{t-1}+\nu_t) + D_3INV_t^e + \\ D_4E_t(\rho^2 p_{t-1}+\rho\nu_t+\nu_{t+1}) + D_5E_t(INV_{t+1}-INV_{t+1}^e) + \eta_t \end{aligned} \quad \textbf{(A.4)}$$

While they know the exact value of the LHS of this equation, speculators will form an expectation about all the terms appearing on the RHS based on I_{t+1}. To keep the calculations manageable we will introduce some bounded rationality on the part of the speculators by assuming the following:

$$E(E_t(INV_{t+1} - INV^e_{t+1})|I_{t+1}) = 0 \tag{A.5}$$

Using this expression we can rewrite equation **(A.4)** as follows:

$$\begin{aligned} INV^R_t - g(t) &= (D_2 + \rho D_4)\nu_t + \eta_t \\ \textit{where}\quad g(t) &\equiv (D_1 + \zeta D_6)\varepsilon_t + (D_2 + \rho D_4)\rho p_{t-1} + D_3 INV^e_t \end{aligned} \tag{A.6}$$

All the terms in g(t) are incorporated into I_{t+1} but, by contrast, speculators cannot decompose the RHS of **(A.6)** into its constituent shocks. It will be shown later that $E_t(INV_{t+1}-INV_{t+1}^e)$ actually depends on ν_t (see Appendix D). So by introducing a limited degree of bounded rationality (equation **(A.5)**) we prevent speculators from recognizing that the LHS of **(A.6)** is also indirectly affected by ν_t through $E_t(INV_{t+1}-INV_{t+1}^e)$. From **(A.6)** the optimal forecast for ν_t can be obtained by using a Kalman filter:

$$E(\nu_t|I_{t+1}) = \frac{\theta}{(D_2 + \rho D_4)}[INV^R_t - g(t)] \quad \textit{where} \quad \theta \equiv \frac{(D_2 + \rho D_4)^2\sigma^2_\nu}{(D_2 + \rho D_4)^2\sigma^2_\nu + \sigma^2_\eta} \tag{A.7}$$

Plugging equation **(A.7)** into **(A.3)**, using the fact that $INV^R_t = INV_t + \eta_t$ and taking the first order condition with respect to INV_t then yields:

$$\frac{\partial INV^e_{t+1}}{\partial INV_t} = \frac{\rho\theta}{1 - D_3} \tag{A.8}$$

Appendix B: Proof of the existence of D_4 and calculation of boundaries for this coefficient.

From equation **(6.7)** in the main text we can derive the following:

$$\frac{\partial F(D_4)}{\partial D_4} = - \frac{2\beta\varphi\delta\rho^2 k^2 \sigma_v^2 \sigma_\eta^2}{[k^2((D_2+\rho D_4)^2\sigma_v^2+\sigma_\eta^2)]^2}(D_2 + \rho D_4) \qquad \textbf{(B.1)}$$

Furthermore, from equation **(6.6)** it is clear that D_4 will be strictly negative if a solution exists. This allows us to draw the following conclusion from equation **(B.1):**

$$\frac{\partial F(D_4)}{\partial D_4} \begin{cases} < 0 & \textit{if} \quad D_4 > -\dfrac{1}{\rho}\dfrac{\varphi\delta}{(k^2+\varphi\delta^2)} \\ = 0 & \textit{if} \quad D_4 = -\dfrac{1}{\rho}\dfrac{\varphi\delta}{(k^2+\varphi\delta^2)} \\ > 0 & \textit{if} \quad D_4 < -\dfrac{1}{\rho}\dfrac{\varphi\delta}{(k^2+\varphi\delta^2)} \end{cases} \qquad \textbf{(B.2)}$$

An examination of the function $F(D_4)$ yields:

$$F(0) = -\frac{\beta\varphi\delta\rho}{k^2}\frac{D_2^2\sigma_v^2}{D_2^2\sigma_v^2+\sigma_\eta^2} \equiv \gamma D \quad \textit{where} \quad 0 \leq \gamma \leq 1$$

$$F\left(-\frac{1}{\rho}\frac{\varphi\delta}{(k^2+\varphi\delta^2)}\right) = 0 \qquad \text{(B.3)}$$

$$\lim_{D_4 \to -\infty} F(D_4) = -\frac{\beta\varphi\delta\rho}{k^2} \equiv D$$

Equations **(B.2)** and **(B.3)** can be summarized by the following picture:

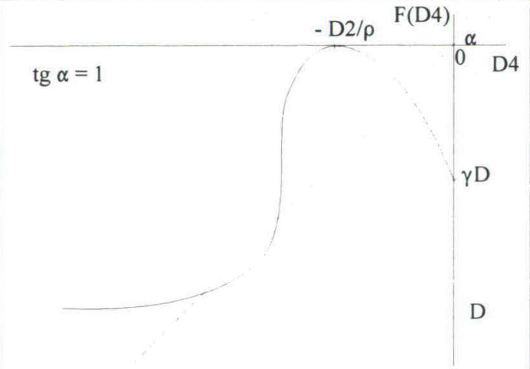


Figure 6.2: Equilibrium solution for D_4

Hence, the function $F(D_4)$ is monotonically increasing on the interval $(-\infty, -D_2/\rho)$. Within this interval there are two possibilities. First of all, it could be that $F(D_4)$ never intersects the 45°-line in which case there is no solution for D_4 on this interval. Alternatively, there could also be two intersections (as shown in Figure 2) yielding two solutions in the range under consideration[91]. Next, as far as the interval $(-D_2/\rho, 0)$ is concerned, one can observe that $F(D_4)$ is strictly decreasing in D_4 in this part of its domain. Furthermore, since $F(D_4)$ reaches a maximum at $D_4 = -D_2/\rho < 0$ and since $F(0) = \gamma D < 0$, it must be that there exists one and only one solution for D_4 on the interval $(-D_2/\rho, 0)$.

These considerations lead us to make the following assumption concerning the boundary conditions for D_4:

$$-\frac{1}{\rho}D_2 < D_4 < 0 \tag{B.4}$$

The reason for selecting this interval is twofold. First of all, we will ensure existence of an equilibrium value for D_4 by choosing the latter in the range described by equation **(B.4).**

[91] Of course it could also be the case that the 45°-line through the origin is at some point in the range specified exactly equal to the slope of the $F(D_4)$-curve, in which case there exists exactly one solution on the interval $(-\infty, -D_2/\rho)$.

Secondly, it is the only possible solution for D_4 for which it holds that the central bank does not display a 'perverse' response to a change in preferences, i.e. for which it holds that following partial derivative of the central bank's reaction function **(6.9)** is positive:

$$\frac{\partial INV_t}{\partial p_t} = D_2 + \rho D_4 \tag{B.5}$$

Appendix C: Derivation of the reaction function of the speculators

Equation **(6.10)** in the main text can easily be obtained as follows: First of all, we plug equation **(A.6)** into **(A.7)** and use the resulting expression in **(A.3)** to get an expression for INV_{t+1}^e in terms of exogenous variables and undetermined coefficients only. Subsequently, we can replace the latter by using the expressions obtained in equation **(6.6).** Lagging the result by one period yields equation **(6.14).**

Appendix D: Derivation of an expression for $E_t(INV_{t+1}-INV_{t+1}^e)$

Taking expectations conditional on the speculators' information set in period t across equation **(6.9)** and subtracting the resulting expression from **(6.9)** we obtain:

$$\begin{aligned} INV_t - INV_t^e &= \left(\frac{\varphi\delta}{k^2+\varphi\delta^2}+\rho D_4\right)(p_t - E(p_t|I_t)) - \\ &\delta D_4[E_t(INV_{t+1}-INV_{t+1}^e) - E(E_t(INV_{t+1}-INV_{t+1}^e)|I_t)] \end{aligned} \tag{D.1}$$

Regarding this equation the following can be noted:

$$\begin{aligned} &E(E_t(INV_{t+1}-INV_{t+1}^e)|I_t) = 0 \\ &p_t - E(p_t|I_t) = \nu_t + \rho(\nu_{t-1} - E(\nu_{t-1}|I_t)) \end{aligned} \tag{D.2}$$

Plugging the expressions obtained in this equation back into equation **(D.1)**, using equation **(A.7)** and leading the result one period yields:

$$INV_{t+1} - INV^e_{t+1} = (\frac{\varphi\delta}{(k^2+\varphi\delta^2)}+\rho D_4)(\nu_{t+1} + \rho(1-\theta)\nu_t - \frac{\rho\theta(k^2+\varphi\delta^2)}{\varphi\delta+\rho D_4(k^2+\varphi\delta^2)}\eta_t) - \delta D_4 E_{t+1}(INV_{t+2}-INV^e_{t+2}) \qquad \textbf{(D.3)}$$

Finally, taking expectations across equation **(D.3)** conditional on the central bank's information set in period t we obtain:

$$E_t(INV_{t+1}-INV^e_{t+1}) = (\frac{\varphi\delta}{k^2+\varphi\delta^2}+\rho D_4)\rho(1-\theta)\nu_t \qquad \textbf{(D.4)}$$

Here, we have used the fact that:

$$E_t(E_{t+1}(INV_{t+2}-INV^e_{t+2})) = E_t(INV_{t+2}-INV^e_{t+2}) = 0 \qquad \textbf{(D.5)}$$

Equation **(D.5)** is a direct result from the observation that the expression on the LHS of equation **(D.1)** is affected by shock realisations in periods t-1, t and perhaps later periods but definitely not by shocks realized in periods t-i , i ≥ 2. Consequently, shocks that are realized in period t (and that are therefore part of the central bank's information set in this period) cannot influence surprise interventions in period t+2.

Appendix E: Proof of Proposition 6.1

The proof for the reaction coefficient for ε_t and the one for p_{t-2} can be seen quite easily by noting that the coefficient D_4 is strictly negative. Furthermore, the proof for the coefficient for ν_{t-1} follows from the fact that the inequality assumed[92] can be reduced to the following:

$$\frac{\varphi^2\delta^3}{k^2}(1-\theta) > \rho D_4\frac{(k^2+\varphi\delta^2)^2}{k^2} \qquad \textbf{(E.1)}$$

[92] By which we mean, for the sake of clarity, the inequality expressing that the coefficient under symmetric information is strictly greater than its counterpart under asymmetric information.

This inequality always holds because the LHS of this equation is greater than or equal to zero while the RHS is strictly negative.

Finally, for the reaction coefficient for v_t the assumed inequality can be rewritten as follows:

$$\frac{\varphi\delta}{(k^2+\varphi\delta^2)} - \delta(1-\theta)(\rho D_4)^2 + \rho D_4 \frac{(k^2+\theta\varphi\delta^2)}{(k^2+\varphi\delta^2)} < \varphi\delta^2 \tag{E.2}$$

Here, the first term on the LHS is strictly smaller than the RHS while the second term is smaller than or equal to zero and the third term is strictly negative.

Appendix F: Proof of Propositions 6.2 and 6.3

From equation **(6.15)** it can be seen that for any given realisation of ε_t , B_t will depend on the coefficient $C \equiv [\ (\varphi\delta+\zeta D_4(k^2+\varphi\delta^2))\ /\ k^2\ ]$, where it holds that $\partial B / \partial C > 0$. Next, from this we can derive:

$$\frac{\partial C}{\partial D_4} = \frac{\zeta(k^2+\varphi\delta^2)}{k^2} > 0 \quad ; \quad \frac{\partial C}{\partial \zeta} = \frac{D_4(k^2+\varphi\delta^2)}{k^2} < 0 \tag{F.1}$$

The proof for the parameter ζ then follows immediately from this equation. As for the parameters β, σ_v^2 and σ_η^2, the proof can be obtained by computing the following partial derivatives from **(6.7)**:

$$\begin{aligned}
\frac{\partial F(D_4)}{\partial \beta} &= -\frac{\varphi\delta\rho(D_2+\rho D_4)^2\sigma_v^2}{k^2((D_2+\rho D_4)^2\sigma_v^2 + \sigma_\eta^2)} && < 0 \\
\frac{\partial F(D_4)}{\partial \sigma_v^2} &= -\frac{\beta\varphi\delta\rho(D_2+\rho D_4)^2\sigma_\eta^2}{k^2((D_2+\rho D_4)^2\sigma_v^2 + \sigma_\eta^2)^2} && < 0 \\
\frac{\partial F(D_4)}{\partial \sigma_\eta^2} &= \frac{\beta\varphi\delta\rho(D_2+\rho D_4)^2\sigma_v^2}{k^2((D_2+\rho D_4)^2\sigma_v^2 + \sigma_\eta^2)^2} && > 0
\end{aligned} \tag{F.2}$$

Figure 2 in Appendix B reveals that a downward (upward) shift in the $F(D_4)$-curve implies a decrease (increase) in the equilibrium solution for D_4, i.e. the signs of the partial derivatives of D_4 with respect to β, σ_v^2 and σ_η^2 are equal to the signs obtained in equation **(F.2)**. The remainder of Proposition 6.2 then follows from combining these results with the fact that $\partial C/\partial D_4 > 0$.

To prove Proposition 6.3 we start by noting that from equations **(6.16)** and **(F.2)** it follows that:

$$\frac{\partial\theta}{\partial\beta} = 2\rho(D_2+\rho D_4)\sigma_v^2\sigma_\eta^2\frac{\partial D_4}{\partial\beta} < 0 \qquad \textbf{(F.3)}$$

Furthermore, to compute the sign of the partial derivatives of θ with respect to σ_v^2 and σ_η^2, we can use the following part of equation **(6.6)**:

$$D_4 = -\frac{\beta\varphi\delta\rho\theta}{k^2} \qquad \textbf{(F.4)}$$

From equation **(F.2)** it can be seen that $\partial D_4/\partial\sigma_v^2 < 0$ and that $\partial D_4/\partial\sigma_\eta^2 > 0$. Since all the other parameters on the RHS of equation **(F.4)** are not affected by these variances, it must hold that: $\partial\theta/\partial\sigma_v^2 > 0$ and that $\partial\theta/\partial\sigma_\eta^2 < 0$.

Appendix G: Proof of Propositions 6.4 and 6.5

On the basis of equations **(6.2)**, **(6.10)** and **(6.11)** the equilibrium solution for Δs_t reads as follows:

$$_t = \alpha + \delta(D_2+\rho D_4)(1-\theta)\rho v_{t-1} - \delta\rho\theta\eta_{t-1} + \delta(D_2+\rho D_4)(1-\delta\rho D_4(1-\theta))v_t + \delta\eta_t + \qquad \textbf{(G.1)}$$

Note that from equation it is immediately clear that both $|\partial\Delta s_t/\partial v_{t\_1}|$ and $|\partial\Delta s_t/\partial v_t|$ will be strictly larger in the presence of ambiguity since its absence implies $\sigma_\eta^2 = 0 \Leftrightarrow \theta = 1$. Together with equation **(6.4)**, **(G.1)** yields the following for $Cov(\Delta s_t, T_t) = E(\Delta s_t T_t) - E(\Delta s_t)E(T_t)$:

$$Cov(\Delta s_t, T_t) = \delta(D_2+\rho D_4)[(1-\theta)\rho^2 + (1-\delta\rho D_4(1-\theta))]\sigma_v^2 \qquad \textbf{(G.2)}$$

In the absence of ambiguity the term between brackets in equation **(G.1)** will be equal to one. $\mathrm{Cov}(\Delta s_t\ ,\ T_t)|_{\sigma\eta 2>0} > \mathrm{Cov}(\Delta s_t\ ,\ T_t)|_{\sigma\eta 2=0}$ then requires the following inequality to hold:

$$(1-\theta)\rho^2 + (1-\delta(1-\theta)\rho D_4) > 1 \qquad \textbf{(G.3)}$$

Since in the presence of ambiguity it holds that $\theta < 1$ and $D_4 < 0$, this inequality will be satisfied.

Furthermore, from **(G.1)** $\mathrm{Var}(\Delta s_t) = E(\Delta s_t)^2 - (E(\Delta s_t))^2$ reads as follows:

$$\begin{aligned} Var(\Delta s_t) &= \delta^2 E(INV_t^R - INV_t^e)^2 + \sigma_\varepsilon^2 \\ &= \delta^2\ [\ (D_2+\rho D_4)^2[(1-\theta)^2\rho^2 + (1-\delta\rho D_4(1-\theta))^2]\sigma_v^2 + (1+\rho^2\theta^2)\sigma_\eta^2\ ] + \sigma_\varepsilon^2 \end{aligned} \qquad \textbf{(G.4)}$$

where $E(INV_t^R - INV_t^e)^2$ provides a natural measure of intervention uncertainty. To prove that $\mathrm{Var}(\Delta s_t)|_{\sigma\eta 2>0} > \mathrm{Var}(\Delta s_t)|_{\sigma\eta 2=0}$ as a result of $E(INV_t^R - INV_t^e)^2|_{\sigma\eta 2>0} > E(INV_t^R - INV_t^e)^2|_{\sigma\eta 2=0}$, it sufficient to show that:

$$(1-\theta)^2\rho^2 + (1-\delta(1-\theta)\rho D_4)^2 > 1 \qquad \textbf{(G.5)}$$

which is satisfied for exactly the same reasons as in equation **(G.3)**.

Chapter 7: Summary and Conclusion

In this book we have looked at the strategy of inflation targeting and its implications for interest rate policy. In the past, the debate about which framework for monetary policy is most appropriate divided economists into two camps: those who favoured non-feed back time-invariant monetary policy rules and those who argued for a discretionary policy which can react to particular economic circumstances. Rules which imply an 'automatic monetary policy pilot' have the advantage of providing a clear nominal anchor by tying down inflationary expectations but have the disadvantage of not being able to deal with shocks which affect the real economy. This may result in severe variability of real variables over time. On the other hand, a discretionary policy which simply reacts to the events of the day without having a clear strategic forward-looking framework may be able to bring about a certain degree of real stability but this will come at the expense of high and variable inflationary expectations. At first sight both approaches may seem incompatible, however, as eloquently described in Bernanke et al. (1999), inflation targeting provides a framework for monetary policy which combines the advantages of both approaches. First of all, the government instructs the central bank to stabilise inflation around the assigned target, to stabilise certain real variables around their natural rates and also provides a ranking for these objectives (since in the short-run these objectives may exert conflicting demands upon policy). This ensures that in the long run monetary policy will only have a level objective for inflation which provides a clear anchor for expectations.

Secondly, the government grants the central bank instrument independence to reach these objectives. This means that short-term political considerations will not influence the conduct of monetary policy. Moreover, to reach her ultimate policy objectives over the medium term, the policymaker will have to react to shocks hitting the economy. In other words, the rule-based elements embodied in the ultimate objectives of monetary policy will be translated into an endogenous policy reaction function in which the interest rate essentially becomes a function of all the determinants of future inflation and output. Finally, the central bank is held accountable for its performance. Since monetary policy has no level objective for real variables, an obvious yardstick in this respect is whether or not inflation has deviated systematically from its target over a prolonged period of time. However, in the short run accountability can and (probably should) also be enhanced by means of some degree of transparency about the central bank's objectives and/or its information about the economy.

Chapter 2: The effect of inflation targeting on the term structure of interest rates

Because of time lags between changes in the instrument of monetary policy and their effect on the ultimate objectives of monetary policy, the implementation of inflation targeting implies that the conditional inflation forecast based on all information currently available will become the intermediate target of monetary policy (see Svensson (1997b)). The central bank's relative weight on output stabilisation is an important element of this intermediate target since it determines the speed with which inflation is returned to target after the economy has been hit by a (supply) shock. The implication for the short-term interest rate is that a higher relative weight on output stabilisation will cause it to respond less aggressively to the determinants of future inflation and output. The purpose of Chapter 2 is to assess the implications of the implementation of an inflation targeting regime for both short- and long-term interest rates. To this end we amend the Svensson (1997b) model by assuming that aggregate demand is determined by the long-term real interest rate. The latter is related to the central bank's instrument (i.e. the short-term nominal interest rate) through the Pure Expectations Hypothesis (PEH) of the term structure. In this respect it is found that the central bank's relative weight on output stabilisation as well as several parameters governing the structure of the economy may have very different implications for the responsiveness of the short-term and long-term nominal interest rate to economics fundamentals. More specifically, an increase in the relative weight on output stabilisation will reduce the responsiveness of the nominal short-term interest rate but will have an ambiguous effect on the reaction coefficients governing the long-term nominal interest rate. The latter arises because the long real rate and the long-term expected rate of inflation react differently to an increase in this preference parameter. Similarly, the duration of the long-term bond will increase the reaction coefficients in the central bank's optimal instrument rule since it decreases the leverage of the short-term interest rate over the long-term rate. However, an increase in duration will reduce the responsiveness of the long-term nominal rate to economic shocks because it will cause the long-term expected rate of inflation to be less sensitive to economic shocks. Finally, we use this model to study the implications of inflation forecast targeting for the predictive ability of the term spread with respect to future output. In this respect we find that both an increase in duration and a decrease in the relative weight on output stabilisation will enhance this predictive ability.

Chapter 3: What are the macroeconomic effects of interest rate stepping?

A stylised fact of central banking is that interest rates often remain constant for prolonged periods of time in the face of a continuously changing environment. In this chapter we rationalise this practice of interest rate stepping by assuming that the central banker suffers a very small loss in utitlity every time she changes the interest rate. This small 'menu cost' may stem from the fact that the central banker recognises that there are many agents in the economy which are bound into fixed nominal interest rate contracts. Alternatively, frequent interest rate changes may render the central banker vulnerable to allegations of inconsistency or incompetence. The central message of this chapter is that even a very small cost of changing the interest rate will lead to a relatively large band around the inflation target within which the central banker will choose not to change the interest rate. This is because the central banker will take the option value of the status quo into account. In other words, the central banker has an incentive to wait for a small amount of time during which new information may arrive indicating that inflation will return to target of its own accord.

The width of the inflation band is shown to be increasing in the cost of changing the interest rate, the volatility of demand shock and the interest rate sensitivity of aggregate demand and decreasing in the slope of the Lucas supply function. Next, we determine the effect of these parameters on the average size of the interest rate step. Of course, the latter will be an increasing function of the width of the inflation band. However, an increase in the interest rate sensitivity of aggregate demand will have an ambiguous effect on the average size of the step. The effect of the afore-mentioned parameters on the expected time period till the next interest rate step is shown to depend on how close the current rate of inflation is to one of the edges of the band. This is because the impact of these parameters works through the volatility of the process driving inflation in the absence of interest rate changes. An increase in this volatility will increase the width of the band but will also make it more likely that inflation will hit one of its edges in the near future. Finally, we examine the impact of the cost of changing the interest rate on inflationary expectations. It is shown that the economy will suffer from an inflationary bias if the cost of increasing the interest rate exceeds the cost of lowering it.

Chapter 4: Should inflation targeting central banks be conservative in the face of uncertainty about potential output?

The appointment of a central banker who is more inflation averse than society is a time-honoured solution to the inflationary bias problem stemming from the desire to push output above the natural rate systematically (see Rogoff (1985)). At first sight the need to appoint such a conservative central banker may seem to disappear once the latter tries to stabilise output around the *natural* rate. However, Clarida et al. (1999) showed that in the context of a New-Keynesian Phillipscurve welfare can still be improved if the central banker is conservative. The first purpose of Chapter 4 is to investigate whether or not this also holds in a model in which price setting is purely backward looking. To this end we investigate a simplified version of the Svensson (1997b) inflation forecast targeting model and find that the optimal degree of output stabilisation will be strictly lower than society's relative weight in this respect. The reason is that a central banker acting under discretion will not take the effect of her current policy actions on next period's inflation rate into account. We establish that the optimal relative weight on output stabilisation will increase when the slope of the Phillipscurve decreases and/or when society's relative weight on output stabilisation increases. The second purpose of this chapter is to assess the effect of uncertainty about potential output on the optimal degree of conservatism. The idea is that this type of uncertainty may alter the unconditional variances of inflation and the output gap to a different degree. Since the optimal degree of conservatism is essentially determined by the trade-off between these variances, it may be affected by this type of uncertainty as well. In particular, we assume that the central bank does not have a perfect observation on potential output in real time but regularly receives updates about past levels of potential output. When setting monetary policy the central banker will use these updates as well as past inflation rates to optimally predict the current level of potential output. On the assumption that the stochastic process driving potential output is stationary, a rational central banker will therefore not make any *systematic* mistakes in her estimates. This is also the reason why the balance between the *unconditional* variances of inflation and the output gap will not be affected compared to the case where potential output is fixed and known. In other words, the need for a conservative central banker seems to stem from the fact the current inflation rate is a function of the rate of inflation in the previous period rather than from the uncertainty about potential output.

Chapter 5: Does an expectations-augmented convex Phillipscurve yield an additional gain from output stabilisation?

Keynes (1936) already argued that, because of capacity constraints, booms may be more inflationary than recessions are disinflationary. The idea of such a convex Phillipscurve has been revived in recent literature. For instance, Clark et al. (1995) show that an accelerationist Phillipscurve implies that there is an additional social return to output stabilisation since the mean level of output will be negatively related to the variability of output. The intuition behind this is that any positive value of the output gap will have to be matched by a negative value of larger magnitude to return inflation back to target. The central issue in this chapter is to investigate whether or not such an additional return to output stabilisation is also present in the presence of a convex expectations-augmented Phillipscurve. In this case the need to bring about a negative value of the output gap to disinflate the economy will crucially depend on the credibility the policymaker. We start by analysing a linear model in which the central bank aims to stabilise inflation around the assigned target and to stabilise output around the long-term natural rate. The latter ensures that the monetary policy will not be burned by the type of credibility problem which leads to a systematic inflationary bias. However, on the assumption that cost-push shocks are partially anticipated by the public, there appears to be another type of credibility problem for a central banker acting under discretion. Since the central banker's reaction to the cost-push shock will be either partly or fully anticipated (depending on whether or not she has some degree of private information about the realisation of this shock), welfare would be improved if she were able to commit to not reacting to this shock. In other words, a central banker acting under discretion will bring about a suboptimally high variability of inflation. Hence, in the absence of a credible commitment mechanism welfare can be improved by appointing a conservative central banker. We show that the optimal degree of conservatism will be bounded between zero and infinity if the central banker has some degree of private information. Moreover, we also examine the determinants of the optimal degree of output stabilisation which is shown to be positively related to the slope of the Phillipscurve and society's relative weight on output stabilisation and negatively related to the degree of persistence in the process driving cost-push shocks. If the central bank has no private information because of which output stabilisation is impossible, we show that it is optimal to appoint a central banker who only cares about inflation stabilisation.

Next, we analyse the case where the central banker cannot affect output under a convex Phillipscurve. We show that a policy of strict inflation targeting which was shown to be optimal in the linear case will cause a deflationary bias in monetary policy. This is because the central bank essentially hedges against the asymmetric risks surrounding the central inflation forecast. In the case of flexible inflation targeting the long run expected rate of inflation is shown to be increasing in the central bank's relative weight on output stabilisation. This is because both uncertainty about demand shocks and persistent cost-push shocks will cause output to fall below potential on average. If the central bank cares about output stabilisation, it will try to offset these effects. However, since the central banker has no private information, these attempts will be futile and will only cause a ceteris paribus increase in the long run expected rate of inflation. Nevertheless, in a non-linear world this effect implies that there will be a social return to output stabilisation. This additional return does *not* arise because it affects the mean or variance of output but rather because it can be used to render the deflationary bias less severe.

Chapter 6: Opaqueness in the conduct of sterilized foreign exchange intervention

In a world without capital controls monetary policymakers have to choose between either targeting domestic objectives or controlling the nominal exchange. If they opt for the first alternative this does not mean that monetary policy will not be influenced by nominal exchange rate movements since the latter will affect the determinants of domestic objectives. However, for a large economy with a limited degree of openness, the exchange rate will have relatively little effect on interest rates. Moreover, central bankers usually employ sterilised foreign exchange interventions as a first line of defense in the occurrence of a misalignment. It is generally agreed that these interventions are likely to derive most of their effect from the fact that they convey new information to the markets. Hence, in order to be effective the central bank will need an information advantage. In this chapter we assume that this information advantage comes from two distinct sources. First of all, the central bank has private information about its own preferred value of the exchange rate and, secondly, the central bank injects a certain degree of ambiguity by providing the market with a noisy signal of the actual intervention volume. Following the empirical evidence on short-term exchange rate movements we assume that the exchange rate follows a random walk and that interventions will only be effective to the extent that they are unexpected. Extending the model developed by Almekinders (1996) we find that repeated interaction between the central bank and speculators will reduce the reaction coefficients in the intervention reaction function compared to the case where interaction takes place only once. However, policy will still suffer from an intervention bias in the sense that part of the central bank's transactions will be expected ex ante and will hence be uneffective. This intervention bias will decrease when the central bank's discount factor increases, when central bank preferences become more volatile over time, when the degree of ambiguity diminishes or when the degree of exchange rate persistence increases. Next, we analyse the impact of ambiguity from a positive perspective and find that providing a noisy signal of the intervention volume will increase both the covariance between exchange rate movements and the central bank's target and the variance of exchange rate movements.

Inflation targeting is a relatively new monetary policy framework which combines some of the lessons gained in practical policymaking during the previous decades and time-honoured and robust results from the academic literature. This book has attempted to contribute to the latter. In this respect we had to leave many issues aside to answer some questions and many issues are still unresolved. Finally, from a practical perspective it is unlikely that inflation targeting represents 'the end of history' as far as an appropriate framework for monetary policy is concerned. Rather, we think it should be seen as an important step forward along the learning curve. Doubtlessly, a rapidly changing world will continue to yield challenges for practical policymakers and raise interesting issues for academic economists in the future.

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Samenvatting (Summary in Dutch)

In dit boek worden de strategie van een directe inflatiedoelstelling alsmede de implicaties hiervan voor het rentebeleid geanalyseerd. In het verleden verdeelde het debat over welk monetair raamwerk het meest gepast is economen in twee kampen: aan de ene kant diegenen die een voorkeur hadden voor een vaste monetaire beleidsregel zonder terugkoppeling en aan de andere kant diegenen die pleitten voor een discretionair beleid dat op specifieke economische omstandigheden kan reageren. Regels die als 'automatische monetaire beleidspiloot' functioneren hebben het voordeel dat ze een duidelijk nominaal anker met zich meebrengen vanwege het feit dat ze als vast referentiepunt voor inflatieverwachtingen dienen. Deze regels hebben echter als nadeel dat ze niet kunnen reageren op schokken die de reële economie beïnvloeden hetgeen kan resulteren in ernstige variabiliteit van reële variabelen. Aan de andere kant zou een discretionair beleid, dat simpelweg reageert op de toestand van het moment zonder een duidelijk strategisch vooruitziend raamwerk, wellicht in staat zijn een bepaalde mate van reële stabiliteit teweeg te brengen. Dit zal echter ten koste gaan van hoge en volatiele inflatieverwachtingen. Op het eerste gezicht lijken beide benaderingen wellicht niet verenigbaar. De strategie van een directe inflatiedoelstelling biedt echter een monetair beleidsraamwerk dat de voordelen van beide benaderingen combineert. Ten eerste instrueert de overheid de centrale bank om de inflatie rond de door haar vastgestelde doelstelling te stabiliseren, om bepaalde reële variabelen rond hun natuurlijke niveaus te stabiliseren en de overheid geeft ook een rangorde voor deze doelstellingen aan (dit is van belang omdat deze doelstellingen op de korte termijn strijdig kunnen zijn). Dit zorgt ervoor dat monetair beleid op de lange termijn enkel het niveau van de inflatie bepaalt hetgeen een duidelijk anker voor inflatieverwachtingen verschaft.

Ten tweede verschaft de overheid de centrale bank instrument onafhankelijkheid om deze doelstellingen te realiseren. Dit betekent dat korte termijn politieke overwegingen geen invloed zullen uitoefenen op de implementatie van het monetaire beleid. Om op de middellange termijn de beleidsdoelstellingen te kunnen realiseren zal de centrale bankier moeten reageren op economische schokken. Hierdoor zullen de, op regels gebaseerde, uiteindelijke monetaire beleidsdoelstellingen worden vertaald in een endogene beleidsreactiefunctie waarin de rente in essentie een functie is van alle determinanten van de toekomstige inflatie en de toekomstige productie. Ten slotte dient de centrale bank democratische verantwoording af te leggen voor het door haar gevoerde beleid. Omdat het monetaire beleid nastreeft reële variabelen rond hun natuurlijke niveau te stabiliseren biedt de vraag of de inflatie al dan niet systematisch is afgeweken van de officiële doelstelling een voor de hand liggende maatstaf in dit opzicht. Echter, op de korte termijn kan de mate van beleidsverantwoording ook worden versterkt middels een zekere mate van transparantie aangaande de doelstellingen van de centrale bank en/of de informatie die ze heeft over economische ontwikkelingen.

Hoofdstuk 2: Het effect van een directe inflatiedoelstelling op de rentetermijnstructuur

Vanwege het bestaan van vertragingen tussen veranderingen in het monetaire beleidsinstrument en het effect hiervan op de uiteindelijke doelstellingen van het monetaire beleid, impliceert de implementatie van de strategie van een directe inflatiedoelstelling dat de conditionele inflatieverwachting (gebaseerd op alle beschikbare informatie) als intermediaire doelstelling van het monetaire beleid zal fungeren. Het relatieve gewicht dat de centrale bank toekent aan the stabiliseren van de productie vormt een belangrijk onderdeel van deze intermediare doelstelling. Dit komt omdat deze bepalend is voor de snelheid waarmee de inflatie in de richting de officiële doelstelling zal worden teruggeleid nadat een economische (aanbod) schok is opgetreden. Voor de korte rente betekent dit dat een hoger relatief gewicht op stabilisatie van de productie ervoor zorgt dat deze minder activistisch op de determinanten van de toekomstige inflatie en productie zal reageren. Hoofdstuk 2 beoogt de implicaties van de implementatie van de strategie van een directe inflatiedoelstelling voor de rentetermijnstructuur te analyseren. In dit verband breiden we het model van Svensson (1997b) uit door aan te nemen dat de geaggregeerde vraag wordt bepaald door de reële lange rente. Deze laatste is gerelateerd aan het instrument van de centrale bank (de nominale korte rente) via de Pure Verwachtings Hypothese van de rentetermijnstructuur. In dit verband vinden we dat het relatieve gewicht op stabilisatie van de productie alsmede verschillende andere parameters die de structuur van de economie bepalen hele verschillende implicaties kunnen hebben voor de mate waarin korte en lange rentes op onderliggende schokken reageren. Meer specifiek zal een toename in het relatieve gewicht op stabilisatie van de productie de gevoeligheid van de korte rente voor deze schokken reduceren. Echter, deze toename zal een ambigu effect hebben op de reactiecoefficiënten van de lange nominale rente. Dit laatste wordt veroorzaakt door het feit dat de lange reële rente en de lange termijn verwachte inflatie verschillend reageren op een toename in deze preferentieparameter. Verder zal een toename in de 'duration' van de lange termijn obligatie de reactiecoefficiënten in de optimale beleidsregel van de centrale bank vergroten. Dit komt omdat een toename in 'duration' het hefboomeffect van de korte op de lange rente vermindert. Echter, deze toename zal tevens de gevoeligheid van de lange nominale rente voor economische schokken verminderen vanwege het effect op de lange termijn verwachte inflatie. Ten slotte wordt het model gebruikt om de implicaties van de strategie van een directe inflatiedoelstelling voor de voorspellende waarde van het verschil tussen de korte en de lange rente voor de toekomstige productie te analyseren. In dit opzicht vinden we dat een toename in 'duration' en een afname in het relatieve gewicht op stabilisatie van de productie deze voorspellende waarde zal vergroten.

Hoofdstuk 3: Wat zijn de macro economische implicaties van het nemen van rentestappen?

Centrale banken houden vaak gedurende een relatief lange periode de rente constant terwijl de economische omgeving voortdurend verandert. In dit hoofdstuk rationaliseren we deze praktijk door aan te nemen dat de central bankier een zeer klein nutsverlies zal lijden op het moment dat de rente veranderd wordt. Deze kleine 'menu kosten' kunnen hun oorsprong vinden in het feit dat de centrale bankier weet dat vele economische agenten gebonden zijn aan vaste nominale rentecontracten. Verder kan het zo zijn dat de centrale bankier beschuldigd zou kunnen worden van incompetentie of inconsistentie indien de rente zeer frequent aangepast wordt. De centrale boodschap van dit hoofdstuk is dat zelfs een zeer klein nutsverlies tengevolge van een renteverandering tot een relatief grote bandbreedte rond de centrale inflatiedoelstelling zal leiden waarin de central bankier de rente niet zal veranderen. Dit komt omdat de centrale bankier de optiewaarde van de status quo in zijn beslissing zal meenemen. Met andere worden, de centrale bankier heeft een prikkel om nog enige tijd te wachten met een renteverandering. Gedurende deze tijd komt nieuwe informatie beschikbaar die er wellicht op zou kunnen wijzen dat de inflatie zich 'vanzelf' weer in de richting van de centrale inflatiedoelstelling beweegt.

De breedte van de inflatieband blijkt toe te nemen als de kosten van een renteverandering toenemen, als de variantie van vraagschokken toeneemt, als de rentegevoeligheid van de geaggregeerde vraag toeneemt en als de hellingshoek van de Lucas aanbodrelatie afneemt. Vervolgens bepalen we het effect van deze parameters of de gemiddelde grootte van de rentestap. Deze zal natuurlijk een toenemende functie zijn van de breedte van de inflatieband. Echter, een toename in de rentegevoeligheid van de geaggregeerde vraag heeft een ambigu effect op de gemiddelde grootte van de rentestap. Het effect van de eerder genoemde parameters op de verwachte tijdsduur tot de volgende rentestap blijkt af te hangen van hoe dicht de huidige inflatie zich bij een van de grenzen van de inflatieband bevindt. Dit komt omdat het effect van deze parameters werkt via de volatiliteit van het proces dat het verloop van de inflatie beschrijft onder de conditie dat de rente niet aangepast wordt. Een toename in deze volatiliteit zal de bandbreedte vergroten maar zal het tevens waarschijnlijker maken dat de inflatie een van de grenzen van deze band zal raken in the nabije toekomst. Ten slotte analyseren we het effect van de kosten van renteveranderingen op inflatieverwachtingen. Hieruit kan geconcludeerd worden dat de economie opgezadeld zal worden met een inflatie die systematisch hoger is dan de officiële doelstelling indien de kosten van een renteverhoging groter zijn dan die van een renteverlaging.

Hoofdstuk 4: Moeten centrale bankiers met een directe inflatiedoelstelling conservatief zijn wanneer er onzekerheid over het potentiële niveau van de productie is?

De benoeming van een centrale bankier die gekenmerkt wordt door een hogere mate van inflatieaversie dan de maatschappij als geheel is een beproefde oplossing voor het probleem van de inflatoire tendentie die onstaat als gevolg van het feit dat de beleidsmaker systematisch probeert de productie boven het natuurlijke niveau te krijgen (zie bijv. Rogoff (1985)). Op het eerste gezicht lijkt de noodzaak voor een benoeming van een dergelijke conservatieve centrale bankier te verdwijnen wanneer deze een productiedoelstelling heeft die gelijk is aan het natuurlijke niveau. Echter, Clarida et. al. (1999) laten zien dat in de context van een Nieuw-Keynesiaanse Phillipscurve de welvaart toch verhoogd zal worden als de centrale bankier conservatief is. De eerste doelstelling van hoofdstuk 4 is te onderzoeken of dit ook geldt in een model waarin het prijszettingsgedrag volledig gebaseerd is op het verleden. We analyseren een vereenvoudigde versie van het inflatie voorspellingsmodel van Svensson (1997b) en concluderen dat het optimale relatieve gewicht op stabilisatie van de productie lager zal zijn dan het maatschappelijk gezien optimale relatieve gewicht. De reden hiervoor is dat een centrale bankier die een discretionair beleid voert geen rekening zal houden met het effect van huidige beleidsacties op de toekomstige inflatie. We stellen vast dat het optimale relatieve gewicht op stabilisatie van de productie zal toenemen als de hellingshoek van de Phillipscurve afneemt en/of als het sociaal gezien optimale relatieve gewicht toeneemt.

De tweede doelstelling van dit hoofdstuk is om het effect van onzekerheid omtrent het potentiële niveau van de productie op de optimale mate van conservatisme te onderzoeken. Het idee hierachter is dat dit type onzekerheid de onconditionele varianties van inflatie en de productie in verschillende mate zou kunnen veranderen. Omdat de optimale mate van conservatisme in principe bepaald wordt door de afruil tussen deze twee varianties, kan deze ook door dit type onzekerheid beïnvloed worden. In het bijzonder wordt aangenomen dat de centrale bankier geen perfecte waarneming heeft van het huidige niveau van de potentiële productie maar dat hij regelmatig revisies krijgt van in in het verleden gerealiseerde niveaus van de potentiële productie. Indien de tijdreeks die het verloop van het potentiële niveau van de productie beschrijft stationair is, zal een rationele centrale bankier geen systematische fouten maken in zijn schattingen van het huidige niveau van de potentiële productie. Daarom zal de afruil tussen de onconditionele varianties van inflatie en de productie ook niet veranderen ten opzichte van het geval waarin het potentiële productie niveau vast ligt en bekend is. Met andere woorden, de noodzaak van het aanstellen van een conservatieve centrale bankier blijkt voort te komen uit het feit dat de huidige inflatie de toekomstige inflatie beïnvloedt en niet uit het bestaan van onzekerheid omtrent het potentiële productieniveau.

Hoofdstuk 5: Impliceert een convexe Phillipscurve met rationele inflatieverwachtingen een additioneel maatschappelijk voordeel van het stabiliseren van de productie?

Keynes betoogde reeds in 1936 dat, vanwege het bestaan van capaciteitsrestricties, een situatie van economische oververhitting de inflatie relatief meer zal verhogen dan dat een recessie deze zal verlagen. De recente literatuur heeft het idee van zo'n convexe Phillipscurve nieuw leven ingeblazen. Clark et. al. (1995) laten bijvoorbeeld zien dat een accelerationistische convexe Phillipscurve impliceert dat er een additioneel maatschappelijk voordeel van het stabiliseren van de productie bestaat. Dit komt omdat het gemiddelde productieniveau dan negatief gerelateerd is aan de variantie van de productie. De centrale vraag in dit hoofdstuk is of een dergelijk additioneel voordeel ook bestaat onder een Phillipscurve met rationele verwachtingen. In dat geval hangt de noodzaak van het induceren van een reductie in de productie om de inflatie te verlagen af van de geloofwaardigheid van de beleidsmaker. Als eerste analyseren we een lineair model waarin de centrale bankier de inflatie stabiliseert rond de officiële doelstelling en de productie stabiliseert rond het lange termijn natuurlijke niveau. Dit laatste zorgt ervoor dat de inflatie niet systematisch hoger zal zijn dan de officiële inflatiedoelstelling. Echter, als we aannemen dat aanbodschokken gedeeltelijk door het publiek voorspeld kunnen worden blijkt er een ander type geloofwaardigheidsprobleem te zijn. Omdat de reactie van de centrale bankier geheel of gedeeltelijk door het publiek geanticipeerd wordt (afhankelijk van de vraag of de centrale bankier al dan niet enige mate van private informatie heeft) zou de welvaart hoger zijn indien de centrale bank zich zou kunnen binden aan de afspraak niet op het voorspelbare gedeelte van de schok te reageren. Met andere woorden, een discretionair beleid zal in dit geval tot een suboptimaal hoge inflatievariantie leiden. Indien een bindende afspraak niet mogelijk is wordt de welvaart daarom verhoogd wanneer een conservatieve centrale bankier benoemd wordt. De optimale mate van conservatisme blijkt tussen nul en oneindig te liggen en blijkt positief gerelateerd te zijn aan de hellingshoek van de Phillipscurve en het maatschappelijk optimale relatieve gewicht op stabilisatie van de productie. De mate van persistentie in de aanbodschok heeft een negatief effect hierop.

Indien de centrale bankier geen private informatie heeft is het optimaal om een beleidsmaker te benoemen die enkel en alleen oog heeft voor het stabiliseren van de inflatie. Vervolgens wordt de casus waarin de centrale bankier de productie niet kan beïnvloeden, en waarin de Phillipscurve convex is, geanalyseerd. Een beleid dat alleen gericht is op inflatiestabilisatie (hetgeen optimaal is als de Phillipscurve lineair is) zal er in dit geval voor zorgen dat de lange termijn gemiddelde inflatie onder de officiële doelstelling zal blijven. In essentie komt dit omdat de centrale bankier zich zal indekken tegen de asymmetrische risico's ronde de centrale inflatieprojectie. Indien de centrale bankier ook aandacht heeft voor het stabiliseren van de productie blijkt de onconditionele inflatieverwachting positief gerelateerd te zijn aan het relatieve gewicht op stabilisatie van de productie. Dit komt omdat zowel het bestaan van onzekerheid over vraagschokken alsmede het bestaan van persistente aanbodschokken ervoor zorgen dat het lange termijn gemiddelde niveau van de feitelijke productie onder het potentiële niveau zal blijven. De centrale bank zal dit trachten tegen te gaan. Echter vanwege de afwezigheid van private informatie zijn deze pogingen tevergeefs en veroorzaken ze slechts een toename in de verwachte inflatie. Desalniettemin zorgt dit effect ervoor dat er in een niet-lineaire wereld zonder private informatie toch een additioneel sociaal voordeel van het stabiliseren van de productie bestaat. Dit additionele voordeel komt niet voort uit het feit dat de centrale bank het lange termijn gemiddelde niveau van de productie kan verhogen indien zij de variantie van de productie verlaagt maar uit het feit dat het ervoor zorgt dat de mate waarin de inflatie systematisch onder de officiële doelstelling blijft verminderd wordt.

Hoofdstuk 6: Niet volledige openheid in de uitvoering van valutamarktinterventies

In een wereld zonder kapitaalrestricties moet de monetaire beleidsmaker kiezen tussen het nastreven van binnenlandse doeleinden of het beïnvloeden van de nominale wisselkoers. Indien men kiest voor het eerste betekent dit niet dat het monetaire beleid niet beïnvloed zal worden door veranderingen in de nominale wisselkoers omdat deze immers tevens invloed heeft of de determinanten van binnenlandse doeleinden. Echter voor een grote economie met een relatief beperkte mate van openheid zal de wisselkoers relatief weinig effect op de nominale rente hebben. In dat geval gebruiken centrale bankiers veelal gesteriliseerde valutamarktinterventies als eerste verdedigingslinie indien zich een wisselkoersonevenwichtigheid voordoet. Veel economen zijn het er over eens dat deze interventies het merendeel van hun effect ontlenen aan het feit dat ze nieuwe informatie aan de markten overbrengen. Dit betekent dat een informatievoordeel essentieel is voor de effectiviteit van gesteriliseerde interventies. In dit hoofdstuk komt dit informatievoordeel uit twee verschillende bronnen. Ten eerste heeft de centrale bank private informatie over de eigen gewenste waarde van de wisselkoers. Ten tweede injecteert ze een zekere mate van ambiguiteit door de markten een met ruis doorspekt signaal van het werkelijke interventievolume te geven. In navolging van vele empirische studies op het terrein van korte termijn wisselkoersbewegingen nemen we aan dat de wisselkoers een 'random walk' (dronkemansloop) volgt en dat interventies alleen effectief zijn voorzover ze niet door de markt verwacht worden. Voortbouwend op een model ontwikkeld door Almekinders (1996) concluderen we dat herhaalde interactie tussen de centrale bank en speculanten ervoor zorgt dat de reactiecoefficienten in de interventiereactiefunctie in absolute waarde lager zullen zijn dan in het geval waar interactie slechts een keer plaatsvindt. Echter, het is nog steeds zo dat een deel van het interventievolume ex ante door de markt verwacht wordt en derhalve geen effect zal sorteren. Deze 'interventie bias' zal lager zijn indien de disconteringsfactor van de central bank toeneemt, indien de preferenties van de centrale bank volatieler worden, als de mate van ambiguiteit afneemt of wanneer de mate van persistentie in wisselkoersbewegingen toeneemt. Vervolgens analyseren we het effect van ambiguiteit vanuit een positieve invalshoek en laten we zien dat het afgeven van een met ruis doorspekt signaal van het interventievolume zowel een toename in de covariantie tussen wisselkoersbewegingen en de wisselkoersdoelstelling enerzijds, als een toename in de variantie van wisselkoersbewegingen anderzijds teweeg zal brengen.

De strategie van een directe inflatiedoelstelling is een relatief nieuw raamwerk voor monetair beleid dat enkele lessen die getrokken zijn uit de praktische uitvoering van dit beleid combineert met robuste resultaten uit de academische literatuur. Dit boek heeft een poging gewaagd om een bijdrage te leveren aan het laatste. In dit opzicht moesten we vele interessante vraagstukken links laten liggen om ons in staat te stellen enkele vragen te beantwoorden. Echter, er zijn nog vele zaken die om verder onderzoek vragen. Tenslotte, kijkend vanuit een praktische perspectief, lijkt het onwaarschijnlijk dat de strategie van een directe inflatiedoelstelling 'het einde van de geschiedenis' zal zijn in de zoektocht naar een passend raamwerk voor monetair beleid. Het moet wellicht veleer gezien worden als een belangrijke stap voorwaarts langs de leercurve. Ongetwijfeld zal een snel veranderende wereld in de toekomst steeds nieuwe uitdagingen blijven vormen voor beleidsmakers en zal deze interessante vragen op blijven leveren voor academische economen.

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This thesis contains a collection of papers on issues in inflation targeting and its implications for the way interest rates are set. In this respect, the first part deals with two largely positive issues: the effect of inflation forecast targeting on the term structure of interest rates and the implications of the well-established practice of interest rate stepping for several macroeconomic variables. Part II of the thesis takes a more normative approach and studies the socially optimal relative weight on output stabilisation. First, this is done in a sticky price model where the central banker faces uncertainty about potential output. Next, a model in which goods market prices are flexible is studied under the condition that the policymaker faces a non-linear supply side constraint. The final part of the thesis studies the effect of opaqueness in the conduct of sterilised foreign exchange interventions.

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